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DYNAMICS OF GULF STREAM MEANDERS

Report prepared by: Takashi Ichiye

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ABSTRACT

Hydrodynamical theories on meanders of the Gulf Stream and similar western boundary currents are critically reviewed. In Section 1, observed facts of meanders east of Cape Hatteras are described including data obtained in Operation Cabot and Gulf Stream'60. In Section 2, long waves along a shearing horizontal current in a rotating barotropic system are discussed, based on Haurwitz and Panofsky's model. In Section 3, stability of long waves in a uniform current of a two-layer ocean is treated from Stommel's and Iida and Ichiye's theory. In Section 4, theories on waves on a shearing current in the upper layer of a two-layer including Duxbury, Iwata and Yasui are presented. In Section 5, general criteria on stability of a baroclinic flow are discussed, including Pedlosky's application of Lin's theorem. In Section 6, the Rossby waves in a barotropic and baroclinic ocean are explained from various authors including Fofonoff, Moore, Il'in and Kamenkovich, Welander and Ichiye and instability of waves in an eastward flow is discussed as a cause of meanders. In Section 7, applications of Rossby's constant absolute trajectory theory n to meanders by Saint-Guily, Warren and Ichiye are presented and Greenspan's criterion on stability of a western boundary current is derived. In Section 8, the cross current momentum eddy transfer determined by Webster and Ichiye for the Gulf Stream and the Kuroshio, respectively and the eddy heat transfer determined by Oort for the Gulf Stream are discussed. In Section 9, possibility of testing some theories on meanders by use of a rotating tank is discussed on the basis of Faller's explanation of similarity between β-effect and variable depth. In the appendix, the observed facts of meanders of the Kuroshio are presented from results of Masuzawa, Moriyasu, Uda and Ichiye.

1. INTRODUCTION

One of the most interesting features of the Gulf Stream is its meandering which seems to resemble in some respects the meanders of a river in its flow pattern. Iselin (1936) noticed that the horizontal distribution of isotherms and dynamic topography of the western North Atlantic indicate wavy patterns along the path of the Gulf Stream to the south of Grand Banks after the Gulf Stream leaves the continental shelf. Church (1937) confirmed that the position of the Gulf Stream between Montauk Point and Bermuda is highly variable from surface temperature data obtained with thermograph installed on liners.

Observational techniques for study of meanders of the Gulf Stream were substantially improved by introduction of BT, GEK, Sea Sampler and STD Recorder together with Loran which provides the necessary accuracy in navigation. These instruments have made it possible to measure respectively the temperature in the upper mixed layer and surface currents on board a ship moving with a cruising speed, and therefore, to collect the data quasi-synoptically. Ford and Miller (1952) showed that the Gulf Stream meanderings developed into a large anti-cyclonic loop of about 150 nautical miles diameter in the area between 62° to 66° W and 38° to 41° N.

These observations were taken by one ship on the edge of the Stream for 1050 miles between Cape Hatteras and 63° W from November 15th to December 6th, 1948, using these new techniques and instruments. Besides, they could show that within a period of two weeks the mean axis of a 150-mile segment of the Stream between 66° and 68° W changed its direction by 90°,

suggesting that the propagation speed of the meander crest (northward protrusion) was about 11 n. miles per day.

However, the first extensive survey of the Gulf Stream deploying almost simultaneously six research vessels equipped with these instruments, was made between Cape Hatteras and the Grand Banks during June 6-23, 1950 and is called "Operation Cabot" (Fuglister and Worthington, 1951). During "Operation Cabot", almost daily synoptic plots of a mean temperature of the upper 200-meter layer were made with BT data. These data indicate that the crest of the first wave near 72° W in meanders of the Gulf Stream moved toward the east at a rate of approximately 11 nautical miles a day, as shown in Fig. 1. Also, in this operation it was observed that a large cyclonic eddy consisting of cold water was formed at about 59-61° W and 37-40° N when a large loop in the Gulf Stream became cut off and separated from the Stream in about nine days. The cut-off eddy was elongated in the direction of west northwest to east southeast with a long axis of about 180 n. miles and a short axis of about 70 n. miles. It was not known from this operation how long it took for the trough to develop into a narrow necked loop. Also, the movement and duration of the eddy after its cut-off from the Stream and statistical natures of such eddies such as typical patterns, and place of occurrence were left unanswered by this study.

Fuglister (1951, 1955) proposed a different interpretation of horizontal temperature fields of the Gulf Stream and advanced a hypothesis of multiple currents which consist of a series of overlapping currents and relatively weak countercurrents instead of one continuous, branching current as

heretofore generally accepted. According to this interpretation, an idea of a single current of the Gulf Stream with the meanderings which increased their amplitudes to the east of 65° W is abolished. Instead, it is interpreted that several currents with relatively straight courses flow side by side separated by weak countercurrents and correspondingly, the isotherms run approximately in zonal directions with successive cold and warm belts. In between these two extreme interpretations, there is a type of analysis which retains an idea of a single current but manages to avoid violent and improbable meandering by introducing eddies. Fuglister applied these types of analysis to the temperature field at a 200 m depth of the Gulf Stream between 75° and 52° W observed by three ships in August, 1953 and of the Kuroshio between 140° and 165° E observed by a multiple ship survey in August, 1933.

The idea of multiple currents proposed by Fuglister was based on temperature data alone and the overlapping segments were described as "shingles" from the appearance of the corresponding isotherms. Von Arx and others (1955) found that the shoreward edge of the Gulf Stream between Florida and 70° W was not a continuous front but consisted of shingles with lengths from 100 to 500 km. They tracked the shingles both with a ship and an aircraft which was flying at an altitude of 1500 feet, scanning the sea surface with a radiation thermometer. These shingles were easily recognized by the strong temperature gradient across them, the change of sea state and sea color and the accumulated Sargassum and flotsam. In each segment these characteristics became fainter as observations were pursued downstream and eventually disappeared, but always another frontal segment was encountered at about 90° left. Particularly, visual observations from

the aircraft indicated that a fine shingle structure interrupted the frontal segment at an interval of about 10 km and usually was left-handed. They proposed a possible interpretation of the shingle structure as an effect of the diurnal tide of the Gulf of Mexico which supplies the additional water transport of 10×10^6 m^3/ sec to the Straits of Florida during a twelve-hour ebb period and withholds the same amount during a flood period, since the tidal waves enter the Gulf through the Yucatan Channel and leave it through the Straits of Florida, the averaged outflow through which amounts to 30×10^6 m^3/sec. They speculated that the water entering the Gulf during a flood period is warmed there and flows out into the Florida Current, forming warm blobs in the warm core of the current further north and shingle segments on the shoreward edge. In fact, the core of the Gulf Stream with characteristic velocities of 3 to 6 knots transports a parcel of water for a distance of 130 to 260 km. during a 24-hour period, corresponding to the lengths of shingle segments and blobs. However, the warm blobs which are 50 to 100 km long are commonly located with distances of 100 to 200 km. between centers in the Kuroshio near Japan, although there is no counterpart of the Gulf of Mexico. (Ichiye, 1955). It is yet to be determined whether there is the shingle structure on the shoreward edge of the Kuroshio.

It is considered that the Gulf Stream does not show large meanderings to the south of Cape Hatteras. However, 26 serial sections made at right angles to the mean axis of the Stream off Onslow Bay from the 16th of May through June 7, 1953 indicated that the velocity components lateral to the mean axis of the Stream could change within a matter of a day or two from

a diffluent to a confluent pattern (Von Arx and others, 1955). Serial sections off Onslow Bay were again made by a Woods Hole group from May 20th through June 18, 1958. These data together with those obtained in the Straits of Florida between Miami and Gun Cay by a University of Miami group from 1952 through 1958 were analyzed to determine kinetic energy balance across the Stream by its meanders. (Webster, 1961 a, b). Serial sections of a similar kind were made seventy times along the 30° N parallel off Jacksonville during October and November, 1961 and twenty times off Cape Hatteras during October, 1962 by another Woods Hole group and these results were analyzed by Oort (1964) to determine heat and mass transports across the Stream. These two analyses will be discussed in a chapter below.

In the spring of 1960 for over a period of two and one-half months from the 2nd of April to the 15th of June, an area from Georges Bank off Cape Cod to the east of the Grand Banks and from the continental shelf south to the latitude of Bermuda was surveyed by three Woods Hole research vessels (Fuglister, 1963). These surveys included deep hydrographic sections along the whole length of meridians in the area with spacings of two degrees of longitude and with distances between stations ranging from twenty miles on the continental slope to sixty miles south of the Gulf Stream, tracking neutrally-buoyant floats at depths between 2000 and 4000 m, and transponding surface buoys and measuring the thermal structure to a depth of 150 m with towed thermistors. It was unexpectedly found that during the entire two and one-half months the large meanders changed position by less than the width of the current and that every observed change in the position of the current

could be accounted for by lateral shifts of the Stream with speeds less than 2.5 miles per day. The operation of tracing the position of maximum current along the Stream by use of GEK so that it registers no component normal to such a course indicated, on many occasions, that the path of maximum current was not a single smooth curve but had shingle structure. The feature of the Gulf Stream in this survey showed that the major current flowed in a direction a little north of east without meandering from 69° to 62.5° W, but it turned abruptly northward and formed a large loop, centered around 61.5 °W. Subsequently, the current headed due south for a distance of over 200 miles along 60.5°W forming a "sock" or a cyclonic loop along the east side of which the major current turned to the north again from 37° N, 60.5°W to 41.5° N, 53° W. An eddy with cold water mass seemed to form at the 'toe' of the "sock" or the southwesterly end at the beginning of the survey, moving slowly to the north along an anti-cyclonic curve and one month later this eddy was observed with its center at 36° 50' N; 64° 30' W, elongating from west northwest to east southeast with a major axis of 200 miles and a minor axis of fifty miles. To the east of the "sock", the current formed minor meanderings whose crests and troughs were located at 41.5° N., 57.5° W; 38.5° N, 55° W; 40° N, 54° W and 39° N, 52.5° W, respectively. Fuglister (1963) indicates that the Gulf Stream meanders are not a series of waves gradually increasing in amplitude from west to east, but rather a quasi-stationary pattern with an abrupt increase of the amplitude near 62° W from the data of various cruises since 1947 in an area between 60° and 68° W. (Fig. 2 a and b)

2. Barotropic Models

The first attempt to explain meanders of ocean currents from hydrodynamic theories was almost simultaneously made by Haurwitz and Panofsky (1950) for the Gulf Stream and by Ichiye (1950) for the Kuroshio. Both theories are based on the model of waves along the boundaries of a jet on a barotropic rotating fluid, either with a discontinuous horizontal shear (Haurwitz and Panofsky) or with a discontinuous horizontal velocity (Ichiye). Since the velocity profile assumed by the former authors is more realistic, their result is discussed here and referred to as the H-P theory.

The basis for the H-P theory is observed facts that the amplitude of Gulf Stream meanders begins to increase after the Stream leaves the shelf north of Cape Hatteras, and that on the coastal side of the Stream the velocity increases to its maximum considerably stronger than decreases on the other side, forming the "shear zone" on the former and the "transition zone" on the latter side. The theory was developed in order to explain that the effect of the coastal barrier is to stabilize lateral waves which are perturbations of the shear zone on the edge of the stream. In the H-P theory, the motion is assumed to be independent of depth because the shear zone is so steep that the perturbations are essentially horizontal and thus, the stabilizing effect of gravity may be neglected. The effect of the earth's rotation is considered but not its variation with latitude since the curvatures of the velocity profile across the current are large compared with the latter.

The equations of motion and continuity for the perturbations superposed on the horizontal current U in the x- direction become (after omission

of second order terms)

$$\partial u/\partial t + U\partial u/\partial x = (f - \partial U/\partial y) - (1/\rho)\,\partial p/\partial x \qquad (1)$$

$$\partial v/\partial t + U\partial v/\partial x = -fu - (1/\rho)\,\partial p/\partial y \qquad (2)$$

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (3)$$

It is assumed that the basic current U is geostrophic and thus, variable in a y- direction only.

The boundary conditions are that $v = 0$ at rigid boundaries and that at internal boundaries where U is discontinuous

$$(\partial/\partial t + U\partial/\partial y)(p - p') + v\partial(P - P')/\partial y = 0 \qquad (4)$$

and an analogous equation in U' and v' where a prime indicates quantities on the other side of the boundary and P is the pressure for the basic current. (When U is continuous, v and p must also be continuous across the internal boundary). All the perturbation quantities are assumed to be of form

$$(u, v, p/\rho) = (A, B, C)\exp 2\pi(\pm y + x - ct)/L$$

where L is the wave length and c the wave velocity. When the wave velocity is complex, the real part gives the speed of propagation and the imaginary part c_i gives the rate of decrease or increase of the amplitude according to its negative or positive sign. In different zones with a uniform and different basic current U, the constants A, B and C can be determined as a function of one of them from equations of motion (1) and (2) and boundary conditions and the last boundary condition may be satisfied provided that a relation (frequency equation) exists between wave speed and wave length. For example, if there are two zones of different basic currents, the number of constants corresponding to A, B and C is six and there are four equations of motion and three

boundary conditions.

Three types of velocity profiles for the basic current which is bounded by solid barriers on both sides were considered in the H-P theory.

Model 1: The simplest model consists of two bounded zones with speeds of the basic current U_1 and U_2 respectively, of width d_1 and d_2 (Fig. 3a). The wave velocity is given by

$$c = (U_1 a_1 + U_2 a_2)/(a_1 + a_2) \pm i(U_1 - U_2)/(\sqrt{a_2/a_1} + \sqrt{a_1/a_2}) \quad (5)$$

Here and in the following equations $a_n = \coth(2 d_n/L)$. It is seen that the waves are always unstable. Also, the imaginary part of c is the largest when a_1 and a_2 equal unity. This condition is almost satisfied when the discontinuity occurs more than half a wave length from both boundaries. When d_1 or d_2 is much smaller than L, the imaginary part tends to zero, indicating that the stability increases when the velocity discontinuity approaches either boundary.

Model 2: For the model which consists of the two currents with speeds U_1 and U_2 separated by a shear zone with the current varying linearly with y (Fig. 3-b), the wave speed is given by

$$c = (U_1 + U_3)/2 + (U_3 - U_1)\, L\,(N - R)/4\pi d_2 M \quad (6)$$

$$+ (U_3 - U_1)\, \frac{1}{2} \left[1 - (N+R)\,L/M\,\pi d_2 + \left\{ L^2 (N-R)^2/M + 4 \right\} / 4\pi^2 d_2^2 M \right]^{\frac{1}{2}}$$

where

$$M = 1 + a_1 a_2 + a_1 a_3 + a_2 a_3, \quad N = a_2 + a_3 \text{ and } R = a_1 + a_2.$$

For long waves, for which $a_2 >> a_1$ and $a_3 >> a_1$, the expression under the square root becomes $\left\{ 1 - \frac{1}{2}\pi + (d_3/d_2)/\pi \right\}^2 - d_3 L^2/(\pi d_2^2)$ which is

negative in most cases, excepting if the shear zone is so close to one solid boundary that d_3/d_2 is nearly equal to zero. This again indicates that the solid boundary stabilizes the perturbations on the shear zone.

Model 3: A better approximation to the jetlike velocity profile of the Gulf Stream by four zones where the current speeds in the two outer zones are constant and between them lie the shear and transition zone in which the velocity gradients are constants, as shown in Fig. 3-c. The frequency equation is a cubic of velocity c but the coefficients are quite complicated. However, in the case that $U_1 = U_2$, a condition which seems to be reasonable since the perturbations to be studied are the meanders along the Gulf Stream, the frequency equation becomes much simpler and is given by

$$c_1^3 A - c_1^2 \left[A + S_3(B-E) + S_2(D-E)\right] + c_1 \left[S_3 E + S_2 D - S_3^2 J - S_2^2 I - S_2 S_3 (J+I-H)\right] + S_2 S_3 (S_2 + S_3 - H) = 0 \qquad (7)$$

where

$$c_1 = (c - U_1)/(U_{2\,3} - U_1)$$

$$A = (a_1 + a_2)(1 + a_3 a_4) + (a_3 + a_4)(1 + a_1 a_2)$$

$$B = 1 + a_1 a_2 + a_1 a_3 + a_2 a_3$$

$$D = 1 + a_2 a_3 + a_2 a_4 + a_3 a_4$$

$$E = (a_1 + a_2)(a_3 + a_4)$$

$$J = a_1 + a_2, \quad H = a_2 + a_3, \quad I = a_3 + a_4$$

$$S_2 = L/2\pi d_2 \quad \text{and} \quad S_3 = L/2\pi d_3$$

Equation (5) indicates that c_1 depends not on the numerical values of the basic current, but on the geometry of the zones only. Since d_1 may be considered as the distance between the Gulf Stream and European continent, it is safely assumed that $d_1 >> L$ or $a_1 = 1$. The unstable waves can develop only if the equation has conjugate complex roots which are possible when its discriminant is positive. The values of the discriminant for different values of d_4 / L are given in the following table in a model with $d_3/L = 1/44$ and $d_2/L = 1/11.4$ corresponding roughly to the Gulf Stream:

d_4/L	∞	1/63	1/630	1/4400	1/6300	0
Disc.	1.523	0.244	0.024	0.006	-0.0005	-0.0030

This table suggests that the decrease of the distance between the shear zone and the continent increases the stability of the current. A more instructive presentation is given by Fig. 4 which shows curves of conjugate roots versus wave length L of Equation (5) for different values of d_4 (Ichiye, 1955, 1956). In this model, $d_2 = 100$, $d_3 = 20$ and $d_4 = 20$ (curve I) and 200 (curve II), all in nautical miles. When the wave length is below a critical value, the conjugate roots of (5) have two real values and the waves are stable. When the wave length exceeds the critical one, the roots become complex and the real and imaginary part are plotted as separate curves in this figure. The real part gives the propagation speed of the disturbance while the imaginary part represents the rate of growth of the amplitude. This figure indicates that the longer the distance between the coast and the shear zone, the smaller the critical wave length at which the instability occurs and the greater the

growth rate of the amplitude of unstable waves. For comparison, by use of a theory developed by Kuo (1949), the real and imaginary parts of the dimensionless wave velocity $(c - U_o / U_m - U_o)$ were computed with a model of planetary waves in a zonal jet which has a horizontal velocity profile, as shown in Fig. 3-d (Ichiye, 1956). In this calculation, the width of the jet was taken as 120 n. miles and the velocities U_m and U_o were taken 1.1 and -0.1 knots, respectively. It is seen that the critical wave length for this model (planetary waves) is much larger than for the H-P model (boundary waves) and also, the imaginary part of the wave velocity or growth rate of the amplitude of unstable waves is much smaller. However, the growth velocity for the planetary wave model has maximum value at the wave length of about 250 n. miles, corresponding to the prevalent meanders in the Gulf Stream and the Kuroshio, while the one for the H-P model increases with the wave length and thus, it has no prevalent wave length.

3. Baroclinic Waves (case of no horizontal shear)

Stommel (1953) considered the long waves in a stratified ocean in which horizontal divergence was dynamically important. In order to simplify the analysis, he abandoned the horizontal shear of the basic current and assumed a two-layer ocean, in which the lower layer is sufficiently deep so that there are no horizontal pressure gradient and current there. The geostrophic current U in the upper layer flows in the x-direction. A cross-stream pressure gradient due to this current is given by

$$fU = -g\,\partial H/\partial y = -g\,(\partial D/\partial y)\,(\Delta\rho/\rho)$$

in which H is the elevation of the free surface, D is the thickness of the upper layer and $\Delta\rho$ is the difference in density of the upper and lower layer.

When small perturbations of u, v in velocity and h in H are assumed to be independent of y, the perturbation equations of motion and continuity may be written in the form:

$$du/dt - fv = -g\,\partial h/\partial x; \qquad dv/dt + fu = 0 \qquad (1)\ (2)$$

and

$$(1 + \rho/\Delta\rho)\,dh/dt + D\,\partial u/\partial x + v\,\partial D/\partial y = 0 \qquad (3)$$

where d/dt is the operator $(\partial/\partial t + U\,\partial/\partial x)$. If the perturbations are all in the form $\exp i\,(kx - \sigma t)$, the frequency equation becomes

$$k^2U^2(1-b)^3 - (f^2 + g'k^2D)(1-b) + f^2 = 0 \qquad (4)$$

where $g' = g\,\Delta\rho/\rho$, $b = c/U = \sigma/kU = b' + b''i$ and b' and b'' are real. Equation (4) can be rewritten as

$$Y^3 + 2 - \delta Y = 0 \qquad (5)$$

where

$$\delta = (2f/kU)^{2/3}\,(1 + g'k^2 D f^2)$$

$$Y = 2\,(2f/kU)^{-2/3}\,(1 - b)$$

The roots of Equation (5) are all real if $\delta > 3$ and have conjugate complex values if $\delta < 3$. As in the previous case, real and conjugate complex roots correspond to stable and unstable waves, respectively. The stability criterion $\delta > 3$ becomes $U^2 < g'D$. When $U^2 = g'D$, the instability begins with vanishing b' or stationary waves, wave number of which is $k = f/\sqrt{2}\,U$. Unstable waves have a narrow range of wave numbers about this value for slightly larger U than the critical value. When numerical values of $D = 200$ m, $U = 2$ m/sec and $\Delta\rho/\rho = 2 \times 10^{-3}$ are used, the critical wave length becomes 180 km., which corresponds to that of the large stationary meander observed in Operation Cabot. Rossby (1951) also showed that the velocity of the Gulf Stream approaches a critical value $(g'D)^{1/2}$ in a steady state due to vertical concentration of momentum, and that the steady state may be broken down if the velocity exceeds this value. However, there is no indication that the depth D of the Gulf Stream decreases sharply between Cape Hatteras and the tail of the Grand Banks and thus, this theory does not explain the sudden increase of the meanders east of about 65° W, as described in the previous section.

Iida and Ichiye (1963) showed that there is no unstable regime for the perturbations in the uniform geostrophic current of a two-layer ocean when the linear increase of D with the lateral distance y is taken into account. Stommel assumed that the thickness of the upper layer D is constant and also its lateral gradient $\partial D/\partial y$ is finite and constant. This

assumption may be permissible in the central part of the ocean, but incompatible along the edge of the Gulf Stream, where the pronounced meanders were observed. When there is a uniform current U in the upper layer, the depth D decreases linearly with y to the left of the current (facing downstream) and thus, the interface between the two layers intersects the surface (Fig. 5). When this intersection is taken as the x-axis, the two layer ocean extends to the negative y-plane and D is expressed by $-sy$, where $-s = fU/g'$. Since the perturbations u, v, and h are dependent on y as well as x, the righthand side of Equation (2) is replaced by $-g\,\partial h/\partial y$. The elimination of u and v in Equations (1) to (3) leads to the equation for h:

$$d(D\,dh/dy)/dy + (fks/\sigma_1 - k^2 D - \Delta/g's)\,h = 0 \tag{6}$$

where $\sigma_1 = \sigma - kU$ and $\Delta = f^2 - {\sigma_1}^2$. Stommel's frequency equation may be derived by equating the coefficient of h in Equation (6) to zero. When $D - sy$ is substituted in this equation, we have

$$d(y\,dh/dy) - \left[k/\omega + f(1-\omega^2)/U + k^2 y\right] h = 0 \tag{7}$$

where $\omega = \sigma_1/f$.

In order that the normal mode of h which is finite at $y = 0$ and $-\infty$, may exist, ω should satisfy

$$k\,\omega^{-1} + f(1-\omega^2)\,U^{-1} = -(2n+1)\,|k| \tag{8}$$

in which n is positive integers. Equation (8) can be rewritten as

$$W^3 + 2 = \delta' W \tag{9}$$

where δ' is given by

$$\delta' = (2/k')^{-2/3}\left[(2n+1)\,k' + 1\right] \tag{10}$$

and $k' = kU/f$ and $W = -\omega\,(k'/2)^{-1/3}$. As before, Equation (9)

has conjugate complex roots when $\delta' < 3$. However, it is seen that for all positive k', $\delta' \geq 3$, where the equality occurs for $k' = 2$ and $n = 0$. It is also proven that the frequency equation has only real roots when there is a constant geostrophic current U' in the lower layer. In this case, k' in Equation (10) should be replaced with $(1 - U'/U)\,k'$. Since the latter quantity is always positive from geostrophic assumption, the same discussion as in the case of motionless lower layer is valid. It is interesting to note that Equation (9) becomes equal to the frequency equation (30) in Reid's paper (1958), when ω and k' are replaced by $-\omega/f$ and $g'sk/f$, respectively. Therefore, mathematical characteristics of roots of Equation (9) are the same as those for the barotropic waves propagating along a coast in a sea with a depth linearly increasing from the coast.

Although Stommel's original solution has an interesting feature for stability of a baroclinic current, it has some inconsistency. In order to discuss the stability problem, we must take a lateral shear of the current into consideration.

4. Baroclinic Waves (model with horizontal shear)

Iwata (1961) and Duxbury (1963) treated the perturbations in a two-layer ocean in which the geostrophic current in the upper layer is not constant, as in Stommel's case, but is variable along the lateral direction. Iwata discussed the solutions for the cases, in which a horizontal velocity distribution is given by different profiles, including a simple jet of a uniform velocity in an infinite motionless fluid and a uniform mean current in a half plane with another motionless half plane. He employed the assumption of the geostrophic perturbation velocities. This assumption is valid when $|\sigma - kU| << f$, where σ and k are frequency and wave number of the perturbations, respectively. On the other hand, Duxbury treated only the case of the uniform mean flow in a half plane bounded by the motionless half plane but without the assumption of geostrophic perturbation velocities, thus covering more general range of the frequency than Iwata.

The equations of motion and continuity for the perturbations in a two-layer ocean with the motionless lower layer of an infinite depth are almost the same as Equations (1) to (3) in Section 3, except $-g\,\partial h/\partial y$ replacing zero in the righthand side of (2). (Hereafter an equation in other sections will be distinguished by the Roman numeral denoting the section; for example, III-1). However, in most of the recent literature, the thickness of the upper layer instead of the elevation of free surface is used as a variable and thus, the perturbation equations are derived here again.

If $\vec{V}(x, y, t)$ denotes the total horizontal components of velocity in a two-layer ocean with the still lower layer of an infinite depth and $H(x, y, t)$ the thickness of the upper layer, the momentum and continuity equations in the vectorial form become

$$\partial \vec{V} / \partial t + \vec{V} \cdot \nabla \vec{V} + 2 \vec{\omega} \times \vec{V} = - g' \nabla H \tag{1}$$

$$\partial H / \partial t + \nabla \cdot \vec{V} H = 0 \tag{2}$$

in which $g' = g \Delta \rho / \rho$ is the "reduced" gravity and $\vec{\omega}$ is the angular velocity of the earth's rotation. The velocity can be resolved into a mean current $U(y)$ directed along the x- axis and perturbations whose components are u and v. With a similar decomposition of $H = D + h$, the linearized perturbation equations become

$$du/dt + (dU/dy - f) v + g' \partial h / \partial x = 0 \tag{3}$$

$$dv/dt + fu + g' \partial h / \partial y = 0 \tag{4}$$

$$dh/dt + \partial (Du) / \partial x + \partial (Dv) / \partial y = 0 \tag{5}$$

where (d/dt) indicates $\partial / t + U \partial / \partial x$. Notice that here h is not the surface elevation but the perturbation of the thickness of the upper layer. The boundary condition along the lateral boundary between discontinuous mean flows corresponding to (II-4) becomes, with neglect of second order terms,

$$(\partial / \partial t + U_j \partial / \partial x)(h_1 - h_2) + v_j \, d (D_1 - D_2) / dy = 0 \tag{6}$$

for $j = 1$ and 2, where the subscripts denote regions with different mean flows. As a first approximation, Equation (6) should be satisfied at $y = y_o$ (constant), which is the y coordinate of the undisturbed boundary of the two mean flows U_1 and U_2. The lateral excursion of the boundary ξ can be expressed in terms of h_1 and h_2 from the condition that

$H_1 = H_2$ at $y = y_o + \xi$. Thus, the first approximation for ξ is given by

$$\xi = (h_1 - h_2) \left[d\,(D_2 - D_1)/dy \right]^{-1} \tag{7}$$

where h_1 and h_2 are evaluated at $y = y_o$.

In Duxbury's model which is shown in Fig. 6a, the uniform current U extends in the region 1 $(y < 0)$, while the region 2 $(y > 0)$ is motionless. The thickness D in the region 1 and 2 is given by $D_o - sy$ and D_o, respectively. The variables u_j, v_j and h_j are of the form

$$(u_j,\ v_j,\ h_j) = \left[X_j\,(y),\ Y_j\,(y),\ Z_j\,(y) \right] \exp i\,(kx - \sigma t) \tag{8}$$

Elimination of X_j and Y_j in Equations (3) to (5) leads to

$$(D_o - sy)\, d^2 Z_1/dy^2 + s\, dZ_1/dy + (K - k^2 D_o + k^2 sy)\, Z_1 = 0 \tag{9}$$

$$d^2 Z_2/dy^2 + m^2 Z_2 = 0 \tag{10}$$

where

$$K = (\omega^2 - f^2)\,(g')^{-1} - fsk\,\omega^{-1} \tag{11}$$

$$m = \left[(\sigma^2 - f^2)\,(g' D_o)^{-1} - k^2 \right]^{1/2} \tag{12}$$

$$\omega = \sigma - kU \tag{13}$$

If the perturbation velocities are assumed to be geostrophic, ω^2 and σ^2 are neglected against f^2 in Equations (11) and (12). The boundary condition (6) which couples functions Z_1 and Z_2 becomes

$$i\,\omega_j\,(Z_1 - Z_2) + s\,Y_j = 0 \tag{14}$$

where

$\omega_1 = \omega$, $\omega_2 = \sigma$ and

$$Y_j = -\,g' i\,(fk\, Z_j + \omega_j\, dZ_j/dy)\,(\omega_j^2 - f^2)^{-1} \tag{15}$$

The solution of (9) which satisfies the condition of finiteness at $y = -\infty$ is given by

$$Z_1 = B_1\, W\,(a,\ z) \tag{16}$$

where

$$W(a, z) = \exp(-z/2)\,\Psi(a, 1; z) \qquad (1$$

in which

$$a = (1/2)(1 - K/sk) \qquad (1$$

$$z = 2k(D_o - sy)/s \qquad (19a$$

and $\Psi(a, 1; z)$ is the Tricomi function (Erdelyi et al, 1953). Equation (16) holds for $z > z_o$, where

$$z_o = 2k D_o/s \qquad (19b)$$

For $y > 0$, the solution of (10) which satisfies the condition of finiteness at $y = \infty$ is given by

$$Z_2 = B_2 \exp(-\mu y) \qquad (20)$$

when m^2 is negative $(= -\mu)$, or when

$$\sigma^2 > f^2 + g' L_o k^2 \qquad (21)$$

In this case, the perturbations in region 1 become the Kelvin-type boundary waves. When m^2 is positive, instead of (20), we have

$$Z_2 = A'_2 \cos my + B_2' \sin my \qquad (22)$$

Duxbury did not discuss the case of positive m^2. However, in this case when there is a coast at a finite distance from $y = 0$, A_2' can be expressed with B_2' from the boundary condition that $v = 0$ at the coast. Then, the subsequent process for obtaining the frequency equation is the same as for solution (20). If there is no coast on the side of positive y, there is no relationship between σ and k, except the inequality (21) with the reverse sign. Then, the problem is to determine the relationships among the incident (oblique) waves and the reflected waves in region 1 and the refracted waves in region 2 (Ichiye, 1963).

Solutions (16) and (20) should satisfy two boundary conditions

(14) at $y = 0$. Then, elimination of B_1 and B_2 leads to the frequency equation

$$\left[\omega\,(\omega^2 - f^2) - g'\,sk\left\{f - 2\omega\, W'(\alpha, z_o) / W(\alpha, z_o)\right\}\right]$$

$$\times\left[\sigma\,(\sigma^2 - f^2) + g'\,sk\,(f - \sigma\, M / k)\right] = \omega\,\sigma\,(\omega^2 - f^2)(\sigma^2 - f^2) \qquad (23,$$

where $W'(\alpha, z)$ denotes dW/dz. Introducing the dimensionless variables

$$\nu = \omega / f, \quad \mu = Uk/f = g'skf^{-2} = F^2 z_o/2 \qquad (24\text{ a, b}$$

where

$$F = U / (g' D_o)^{-1/2} \qquad (24c)$$

Equation (23) can be rendered in the form

$$\left[\mu\nu(CE - 1) + (E - \nu)(\nu + \mu)\left\{(\nu + \mu)^2 - 1\right\}\right]^2$$

$$= \left[\nu(\nu + \mu)(CE - 1)\right]^2\left[\mu^2 - \left\{(\nu + \mu)^2 - 1\right\} F^2\right] \qquad (25)$$

in which

$$\alpha = (1/2)\left[1 - (\nu^2 - 1)\,\mu^{-1} + \nu^{-1}\right] \qquad (26)$$

$$C = 2\alpha - 1 \qquad (27)$$

$$E(\alpha, z_o) = W(\alpha, z_o) / 2W'(\alpha, z_o) \qquad (28)$$

The parameter ν is a relative "Doppler" frequency, μ is a relative longitudinal wave number and F is the internal Froude number. Equation (25) is an eight-degree polynomial in ν, in which the coefficients depend upon α, μ and F, and (26) is a third-degree polynomial in ν. A practical method to obtain the relation between ν and μ is first to determine graphs of ν versus μ for selected values of α from Equation (26). Then, from Equation (25) graphs of ν versus α are determined for particular values of F^2 and α, using the computer. Each graph was

divided along its μ - axis into zones of different mode number (n), which was determined by the number of zeros in argument z of $W(\alpha, z)$ or a transverse wave number. Then, the graphs obtained from Equations (25) and (26) are superposed to determine the admissible values of ν and α for common μ and F, corresponding to each mode number n. Further ν and μ must satisfy condition (21) or its dimensionless form

$$\left[(\nu + \mu)^2 - 1 \right] < (\mu / F)^2 \tag{29}$$

One of the $|\nu| - \mu$ graphs for $F^2 = 10$ is shown in Fig. 6b. These graphs indicate that there are two classes of waves: inertio-gravitational waves with $|\nu| > 1$ (or $|\omega| > f$), and quasi-geostrophic ones with $|\nu| < 1$ (or $|\omega| < f$), each class having different mode numbers. Only the zero-order mode behaves like the quasi-geostrophic class for small μ. The inertio-gravitational waves travel upstream while the quasi-geostrophic waves travel downstream. The former has small wave lengths and it appears to be irrelevant to the meanders. The physical parameters for the Gulf Stream, such as, $D_o = 100$ m, $s = 1.5 \times 10^{-2}$, $\Delta\rho / \rho = 1.1 \times 10^{-3}$, $U = 1.8$ m/sec leads to $F^2 = 3$. Duxbury's analysis indicates that there are stable waves of the quasi-geostrophic class of wave lengths and speeds comparable to those of the Gulf Stream meanders when the internal Froude number (F) exceeds unity, unlike the result of Stommel (1963). Further, Duxbury considered that the Gulf Stream meanders could be generated by the stable modes from three reasons: (1) The disturbances composed of many modes of dispersive character would grow in amplitude downstream by manner of

phase reinforcement (2) An external energy like wind torque would cause resonance coupling with the stable waves which are actually of the normal modes of the system and (3) Variations of D_o and U may produce amplification analogous to the amplification of swell propagating into shallow water. However, his analysis does not prove non-existence of the unstable waves (or complex roots of the frequency equation) in the range of wave lengths and propagation velocity comparable to the actual meanders. The first two possibilities of exciting meanders by stable modes seem to be unreasonable since occurrence of a localized disturbance and an external energy source is random and yet the actual meanders are rather regular in their wave characteristics and their geographical locations. Also, without evaluation of the solutions corresponding to the initial condition of the localized disturbance, the external force or the variable D_o and U, the three possibilities described by Duxbury are mere speculations. As far as the mechanism of the Gulf Stream meanders is concerned, Duxbury's analysis does not give any light on the problem.

Iwata (1961) derived the frequency equation similar to, but simpler than (23) under the assumption of geostrophic perturbations for the same profile of the mean flow as above. He did not solve the frequency equation numerically nor treated it analytically, in general terms. However, he could show that when $z_o >> 1$, the frequency equation becomes

$$\ell^2 - \left[\mu + F^2 (1 + \mu^{-1} \mu_1{}^{-1})/2\right] \ell + \mu \mu_1{}^{-1} = 0 \qquad (30)$$

from the asymptotic expansion of $W(a, z)$ for large z, where

$$\ell = \sigma / (k U - \sigma) = - \sigma / \nu f \qquad (31)$$

$$\mu_1{}^2 = \mu^2 + F^2 \qquad (32)$$

Also, when $z_o << 1$, the frequency equation becomes

$$2\ell^2 + F^2\ell - F = 0 \tag{33}$$

It is easily seen that Equation (30) has conjugate complex roots, while (33) has real roots only. The definition of z_o given by (19b) yields $z_o = 1.3 \times 10^4\,k$ (k in m^{-1}) from the physical constants corresponding to the Gulf Stream ($D_o = 100$ m, $s = 1.5 \times 10^{-2}\ m^{-1}$). Therefore, $z_o = 1$ yields the perturbation wave length of 84 km. This suggests that in the range of wave lengths comparable to the real meanders, the condition $z_o << 1$ is satisfied and thus, the perturbation is stable, as discussed by Iida and Ichiye (1963). In another example, Iwata (1961) treated the case of the jetlike mean flow with a constant velocity and a finite width, as shown in Fig. 6c. When the thickness of the upper layer of the jet is a linear function of y, satisfying the geostrophic relation for the mean flow, the solution for the jet region corresponding to Equation (16) includes the Kummer function $\Phi(\alpha, c; z)$ (Erdelyi et al, 1953) as well as the Tricomi function and the analytical treatment of the resultant frequency equation becomes intangible. Iwata alternatively assumed that the depth of the upper layer of the jet is constant on the basis of smallness of its velocity and obtained the frequency equation which gives unstable waves, under the assumptions that the wave lengths of the disturbances are much smaller than 300 times the width of the jet. His frequency equation is only an approximation, but it is certain that there is a range of unstable waves with wave lengths comparable to the real meanders. Also, validity of his assumption of the uniform depth of the jet must be checked by scaling

basic equations using characteristic quantities as will be discussed below. However, for wave lengths much larger than the width of the jet, this assumption might be valid. This is similar to a case discussed by Ichiye (1951) on propagation of long waves in a channel of variable depth and width connected with another uniform channel. It was shown that the variable channel can be replaced with a uniform channel with averaged values of depth and width of the prototype channel when the wave length is sufficiently long. It will be worthwhile to test the applicability of this result to the quasi-geostrophic waves in a geostrophic jet considered here.

Yasui (1961) treated the perturbations in the same two-layer ocean as in Iwata's and Duxbury's model. He obtained an exact solution of the equation of the surface elevation in terms of a hyper-geometric function for the distribution of the mean flow given by $U = U_o \operatorname{sech}^2 (y/L)$ (Bickley jet), under the assumptions that

$$\sigma \gg kU \quad \text{and} \quad |\sigma^2 - f^2| \gg f\,|dU/dy| \qquad \text{(34 a, b)}$$

The frequency equation in this model is given by

$$d\left[\mu^2 + F^2(1-\kappa^2)\right]^{1/2} = 1 - 2n + \left[1 + d'(2\mu\kappa + \mu/?)\right]^{1/2} \qquad \text{(3}$$

where n is non-negative integers. The dimensionless parameters in this equation as defined by

$$\mu = U_o k/f; \qquad \kappa = \sigma/f \qquad \text{(36 a,}$$

$$d = 2Lf/U_o, \qquad d' = LfD_1/U_oD_o \qquad \text{(37 a,}$$

while the thickness of the upper layer is given by

$$D = D_o + D_1 \tanh(y/L) \qquad (38)$$

Equation (35) becomes a fifth degree polynomial in κ after rationalization.

Yasui did not discuss the equation in detail. Also, the assumptions (34 a, b) may exclude the waves with a low propagation speed corresponding to the meanders. It will be worthwhile to treat analytically the case of the Bickley jet for the low propagation speed. A conclusion immediately obtained from the frequency Equation (35) is that μ, κ should be positive for positive D_1, indicating that the inertio-gravitational waves along a Bickley jet propagate in an opposite direction to the mean flow, as in the case of a shear boundary discussed by Duxbury (1963).

Yasui (1963) also treated the quasi-geostrophic waves in a jet shown in Fig. 6 (c) of a two-layer ocean by assuming the depth of the jet to be constant in a way similar to Iwata's model (1963). He applied his result to fluctuations of subsurface temperature and density with periods longer than a week observed at an ocean weather station T at 29° N and 135° E. He speculated that the fluctuations of periods of 80 to 100 days were due to the stable waves of a quasi-geostrophic mode having a propagation speed close to the mean flow.

5. Hydrodynamic Stability of the Gulf Stream

A general theory of hydrodynamic stability (Lin, 1955) has been applied to the study of perturbations in the atmospheric jet by many authors including Kuo (1949), whose method of studying the stability of a barotropic flow was also used in the problem of ocean current meanders; for instance by Ichiye (1955), as discussed before. Stern (1961) and Pedlosky (1963) treated the stability of a linear flow with a horizontal shear in a two layer ocean.

Stern (1961) discussed the energy balance of a baroclinic jet in a two-layer ocean. The momentum and continuity equations (III - 1, 2) are valid, if the hydrostatic approximation may be defended. Then, the two layer model is equivalent to a single layer system with a "reduced" gravity and with the depth equaling the thickness of the upper layer. (The condition for the validity of the hydrostatic approximation is that $D_o/L << D_b/L << 1$ when $R \leqslant 1$ and $F \leqslant 1$, where D_o, D_b and L are the mean depth of the top layer and bottom layer and the horizontal scale of perturbation, respectively and R and F are the stream Rossby number and the rotational Froude number, respectively, defined by

$$R = U_o/f\,L \quad \text{and} \quad F = f^2\,L^2/\,g'\,D_o \qquad (1\,a, b)$$

Energy equations may be obtained by multiplying the vectorial form combining (III-3) and (III-4) by $D\vec{v}$ and Equation (III-5) by $g'h$, where $\vec{v}$ is the vector of the perturbation velocity (u, v) and U and D are functions of y only. The results are

$$\partial E/\partial t + U \partial E/\partial x + D u v\,(dU/dy) + g' D\vec{v}\cdot\nabla h = 0 \tag{2}$$

$$\partial Q/\partial t + U \partial Q/\partial x + g' h\,\nabla\cdot D\vec{v} = 0 \tag{3}$$

where

$$E = (1/2)\,D\,|\vec{v}|^2 \qquad \text{and} \qquad Q = (1/2)\,g'\,h^2 \tag{4 a, b}$$

are respectively the kinetic and potential energy of the perturbation in the undisturbed depth D. If Equations (2) and (3) are averaged over the entire horizontal plane, the second terms of both Equations (2) and (3) vanish, when E and Q are finite, because U is the function of y only. Also, the average of the last term of Equation (2) equals, but has a reverse sign to the average of the last term of (3), because

$$\overline{h\nabla\cdot D\vec{v}} = \left[h D v_n \right] - \overline{D\vec{v}\cdot\nabla h} \tag{5}$$

from Green's theorem where v_n is the velocity component normal to the boundary and the brackets indicate the integral along the boundary divided by the total area. The quantity in the brackets vanishes, when the v_n vanishes at the x- boundaries and is periodic in x far downstream. The right-hand side of (5) multiplied by g' represents the work done against the gravitational field. Summing averaged Equations (2) and (3), we have

$$\partial(\overline{E} + \overline{Q})/\partial t = -\overline{D u v\,\partial U/\partial y} \tag{6}$$

which indicates that the rate of increase of total energy of the perturbation depends on the Reynolds stress acting on the horizontal shear of the mean flow. Equation (6) implies that the meander theory of Stommel (1953) discussed in Section 2 is untenable, for in his model, $\partial U/\partial y = 0$ and thus, $\overline{E} + \overline{Q}$ is conserved. Also, this equation indicates that the instability is caused only through the shearing mechanism even when there is a conversion

of mean potential energy to the perturbation, owing to the hydrostatic assumption.

Equations (III-1) and (III-2) imply the conservation of the vertical component of potential vorticity

$$\delta(f + Z + \zeta)H^{-1}/\delta t = 0 \qquad (6)$$

where Z and ζ are the relative vorticity of the mean motion and the perturbation, respectively. H is the total depth or the sum of mean depth D and the perturbation h and $\delta/\delta t$ is the two-dimensional substantial derivation. When linearized, Equation (6) becomes

$$d\zeta/dt - (f + Z)\cdot dh/dt + vD\,\partial H^{-1}(f + Z)/\partial y = 0 \qquad (7)$$

where $d/dt = \partial t + U/\partial x$. When introducing the geostrophic approximations:

$$u = -g'f^{-1}\,\partial h/\partial y, \quad v = g'f^{-1}\partial h/\partial y, \quad \zeta = g'f^{-1}\nabla^2 h \qquad (8)$$

and denoting

$$h = \varphi(y)\exp i\,a\,(x - c\,t), \quad c = a + ib \qquad (9)$$

$$P(y) = (f + Z)/D \qquad (10)$$

we then obtain from Equation (7) the eigen value equation

$$\varphi''(y) - \left[a^2 + fP(y)/g'\right]\varphi - DP'\varphi/(U - c) = 0 \qquad (11)$$

where a prime, except to g, denotes differentiation with respect to y. The boundary condition is that φ vanishes at $y = \pm\infty$. Multiplying (11) by the conjugate eigen function φ^* and averaging the result over all values of y, we obtain

$$b\int_{-\infty}^{\infty} DP'(y)\,\varphi\,\varphi^*\left[(U - a)^2 + b^2\right]^{-1}dy = 0 \qquad (12)$$

by equating the imaginary part to zero (Lin, 1955). Equation (11) indicates that if there are unstable modes (i.e. $b > 0$), then $P'(y)$ must change sign within the jet. Therefore, the potential vorticity of the mean flow $P(y)$ has an extrema within the jet as a necessary condition for instability of the jet.

Where $\zeta << f$ or the Rossby number is extremely small, we have

$$D\,P'(y) \cong U\,(U''/U - f^2/g'D) \qquad \text{(13}$$

If U''/U is always finite, then $P'(y)$ never vanishes or the jet is stable whe

$$f^2/g'D_i > \max.\ (U''/U) \qquad \text{(14}$$

where $D_1 = D$ ([illegible]) and max. means the maximum value. Therefore, a jet with a given velocity profile is the more stable, the smaller the depth, if other things being equal.

When D is sufficiently large ($F << 1$), $P(y)$ 0 and $D\,P' \approx dZ/ = U''(y)$. It is easily seen that Equation (11) reduces to the classical eigen value equation of the two-dimensional inviscid disturbance for the mean flow $U(y)$ which is unstable in the vicinity of certain discrete wave numbers (α) from Tollmien's theory (Lin, 1955). Therefore, there must exist a maximum critical depth D_{1c} above which the jet is unstable and the condition for this number is

$$0 < f^2 / (g' D_{1c}) < \max\ U''/U \qquad \text{(1}$$

The more precise criterion requires the solution of Equation (11) which is reduced to the following dimensionless form (if $R << 1$):

$$\phi\,(\xi) - (\alpha_o^{\,2} + F)\,\phi - \left[W''(\xi) - W(\xi) \right] (W - c_o)^{-1}\phi = 0 \qquad \text{(11}$$

where

$$\xi = y/L, \quad U(y) = U_o\,W(\xi), \quad c_o = c/U_o, \quad \alpha_o = \alpha L \qquad \text{(1}$$

In this equation, F is constant to the order of R, since the expansion of the undisturbed depth $D(y)$ is given by $D(y) = D_o + O(\xi R)$ from (IV-38). Whe the mean flow is given by $W = \mathrm{sech}^2\xi$ (Bickley jet), the solution of (11a) becomes $\phi = \mathrm{sech}^2\xi$ provided $\alpha_o^{\,2} = 6\,c_o = (4-F)$. Then long quasi-stationary waves ($\alpha_o \approx 0$, $c_o \approx 0$) become neutral at $F \approx 4$ and thus, unstable solutions exist near the neutral waves from Tollmien's theory

(Lin, 1955). A sufficient condition for the instability of this jet is that $F < 4$. When the physical constants are taken as $f = 10^{-4}\ \mathrm{sec}^{-1}$, $g' = 1\ \mathrm{cm}^2\ \mathrm{sec}^{-1}$ and $L = 50$ km, $D_o = 620$ m corresponds to $F = 4$. When the mean depth exceeds this value, the jet will be unstable provided $R << 1$ or $U << 5\ (= fL)$ m/sec. Stern suggested that the initiation of the Gulf Stream meanders north of Cape Hatteras may be due to the increase of its mean depth.

Pedlosky (1963) discussed the stability of a two-layer system in which the mean flow in the lower layer $U_2(y)$ is finite. Quantities in the upper and lower layers are distinguished with suffices 1 and 2, respectively. The conservation of potential vorticity and the conservation of volume are linearized in each layer by the perturbation method as before, under the conditions of the hydrostatic pressure and geostrophic balance. The non-dimensional perturbation equations for the pressure p_1 and p_2 can be written as

$$(d^2/d\xi^2 - \alpha^2)\, p_i + S_i\, p_i\, (W_i - c)^{-1} + F_i\, (p_1 - p_2)(-1)^i = 0 \quad (i=1,2) \qquad (1$$

where all the quantities are made dimensionless by scaling the variables with (16) (and dropping the suffix o in α and c), F_i is the Froude number defined by (1 b) with D_o replaced by D_i or the undisturbed thickness of the upper and lower layer, and

$$S_i = \beta R^{-1} - W_i''(\xi) - F_i\, H_i'(\xi), \quad (i = 1, 2) \qquad (18)$$

in which β is the non-dimensional rate of change of f with latitudes and a prime indicates the differentiation with ξ. Multiplying each equation of (17) by p_i^* (complex conjugate of p_i) and subtracting from this its conjugated equation, we have

$$(p_i^* p_i' - p_i p_i^{*\prime})' + 2\, b\, S_i\, |p_i|^2\, |W_i - c|^{-2}$$ (19

$$= F_i (-1)^i (p_1 p_2^* - p_2 p_1^*), \quad (i = 1, 2)$$

in which b is the imaginary part of non-dimensional $c (= a + ib)$. Integrating two equations of (18) between $\xi = -1$ and 1, where p_1 and p_2 vanish (the values ±1 may be replaced by any other constants) and summing the results, we have

$$2b \int_{-1}^{1} \sum_{i=1}^{2} D_i S_i\, |p_i|^2\, |W_i - c|^2\, d\xi = 0 \tag{20}$$

Therefore, if $b \neq 0$, S_i must be positive in some regions and negative in others.

One interesting application of Equation (20) is the case where $S_i = 0$ and $W_2 = 0$, neglecting β. Then in the upper layer

$$W''(\xi) - F_1 W_1(\xi) = 0 \tag{21}$$

since $D_1'(\xi) = -W(\xi)$ in non-dimensional units. If there is a wall at $\xi = 0$, the mean flow is given by

$$W_1 = \exp \xi \sqrt{F_1} \qquad (-\infty < \xi < 0) \tag{22}$$

Since $D_2'(\xi) = -D_1'(\xi) = W_1$ and S_2 does not change sign, Equation (19) yields $b = 0$ and the flow is stable. Equation (22) represents the velocity profile of the Gulf Stream obtained by Stommel (1958) with a condition of conservation of potential vorticity. However, if we change the H_2 at bottom to a variable slope of suitable forms, S_2 may be made both positive and negative and b may become finite. Therefore, the effect of a variable bottom slope may change the stability characteristics of the flow without changing the kinematics of the mean flow.

Howard (1961) proposed a method of obtaining criterion for stability of stratified two-dimensional shear flows. This method was applied to the jet in a two - layered system by Pedlosky (1963, 1964a). In the following process, β will be neglected for simplicity. Introducing the functions

$$\psi_i = p_i \, (W_i - c)^{-1} \qquad (i = 1, \, 2) \tag{23}$$

into Equation (19), multiplying each equation by ψ_i^*, integrating between -1 and 1 with ξ and adding the resultant equations, we have

$$\overline{(W_1 - c)^2 Q_1} + \overline{(W_2 - c)^2 Q_2} = \overline{(W_1 - W_2)^2 J} \tag{24}$$

where a bar indicates integration between -1 and 1 and

$$Q_i \, F_i = \left| \psi_i'(\xi) \right|^2 + a^2 \left| \psi_i \right|^2 \tag{25}$$

$$J = \left| \psi_1 - \psi_2 \right|^2 / 2 \tag{26}$$

Therefore, Q_i and J are positive finite quantities. The imaginary and real parts of (26) are respectively

$$\overline{(W_1 - a) \, Q_1} + \overline{(W_2 - a) \, Q_2} = 0 \tag{27}$$

$$\overline{W_1^{\,2} Q_1} + \overline{W_2^{\,2} Q_2} = (a^2 + b^2) \, (\overline{Q_1 + Q_2}) + \overline{(W_2 - W_1)^2 J} \tag{28}$$

When $n_i \leq W_i \leq m_i$, the following inequality holds

$$\sum_i (W_i - m_i) \, (W_i - n_i) \, Q_i \leq 0 \tag{29}$$

Integrating Equation (29) between $\xi = -1$ and 1, and substituting relations (26) and (27), we have

$$\left[(a^2 + b^2) - a \, (m_1 + n_1) + m_1 n_1 \right] (\overline{Q_1} + \overline{Q_2}) + (m_1 + n_1 - m_2 - n_2) \, \overline{W_2 \, Q_2} + (m_2 n_2 - m_1 n_1) \, \overline{Q_2} < 0 \tag{30}$$

in which $\overline{(W_2 - W_1)^2 J}$ is dropped from the righthand side, since it is positive.

When $m_1 + n_1 > m_2 + n_2$ and $m_1 + n_1 > 0$, W_2 in (30) may be replaced by n_2 and the inequality (30) becomes

$$\left[a - (m_1 + n_1)/2 \right]^2 + b^2 \leq (m_1 - n_1)^2/4$$

$$+ (m_1 - n_2)(n_1 - n_2)\, \bar{Q}_2 / (\bar{Q}_1 + \bar{Q}_2) \tag{31}$$

This equation indicates that the complex phase speed lies within a semi-circle as indicated in Fig. 7, in which it is assumed that $m_1 >> m_2$ and $n_1 << n_2$. It implies that the flow is capable of greater growth rates if the radius of the semi-circle is larger (for instance, comparison of the case $n_1 > n_2$ with the case $n_1 < n_2$).

Pedlosky (1963, 1964b) also showed that the theorem of Tollmien (Lin, 1955) is valid for a two-layer model. It is assumed that the velocity of the lower layer is zero and c_s is the value of W_1 at the point where $S_1 = 0$ or the vorticity has an extremum with $\rho = 0$. The K- functions of Lin's method (Lin, 1955) for the two layers are defined by

$$K_i(\xi) = S_i (W_i - c_s)^{-1}, \quad (i = 1, 2) \tag{32}$$

where $W_2 = 0$. If

$$K_1 + K_2 > \pi^2 / (2a^2) \tag{33}$$

where $2a$ is the width of the jet, it is shown that Equation (16) has a neutral solution with phase speed c_s. The proof is given by the variational method for the functional M defined by

$$M = \left[\overline{\sum K_i (p_i)^2 F_i^{-1} - (\partial p_i/\partial \xi)^2 F_i^{-1} - (p_1 - p_2)^2} \right] \Big/ \overline{\sum p_i^2 F_i^{-1}} \qquad (3$$

where a bar denotes the integral with ξ between -1 and 1 as before and the summation is for $i = 1$ and $i = 2$. The functional M has a maximum

where its variations with p_i and p_2, $\delta M / \delta p_i$ and $\delta M / \delta p_2$ vanish. If $c = c_s$, these variational conditions are equivalent to Equation (17). Therefore, a neutral solution p_{is} exists with an eigen value $\alpha_s^2 = \max. M$ for $c = c_s$. It is also shown that the flow becomes unstable to wave numbers slightly less than α_s by use of a method modified from the one developed by Lin (1955). Let p_i be an eigen function for α which is near α_s. Multiplying Equation (17) for p_{is} and p_i, respectively with p_i and p_{is}, summing all of them and integrating with ξ over the range $(-1, 1)$ we have

$$(\alpha^2 \alpha_s^2)/(c - c_s) = \overline{\Sigma k_i p_i p_{is} F_i^{-1} (W_i - c)^{-1}} \Big/ \overline{\Sigma p_i p_{is} F_i^{-1}} \qquad (35)$$

By taking limit $\alpha^2 \rightarrow \alpha_s^2$, $c \rightarrow c_s$, we can show that

$$(d\alpha^2 / dc)_{c \rightarrow c_s} = A + iB \qquad (36)$$

with $B > 0$. Thus, if α^2 decreases slightly, the imaginary part of c becomes positive. It is shown that the sine type jet $\left[(W \sim \cos(\pi \xi / 2a) \right]$ or the Gaussian jet $\left[W \sim \exp(-r^2 \xi^2) \right]$ has a neutral solution and the unstable regime near it.

Theories of stability of an atmospheric zonal flow were reviewed by Phillips (1963) for geostrophic disturbances and also by Miles (1964) for disturbances of more general types. The latter showed that disturbances of sufficiently short wavelength in flows with typical velocity distributions are always unstable but that they have a rate of growth vanishing directly with the wave length. Also, the latter proved that flows with sufficiently small velocities are always unstable for all wave lengths.

6. Rossby Waves as a Cause of Meanders

Charts of surface isotherms of the western North Pacific for each ten-day period have been made by the Japan Meteorological Agency since about 1950 by use of the data collected by merchant ships and fishing vessels. Comparison of the surface isotherms with the current pattern based on dynamic topography and direct measurements with G E K showed that the strong gradient of the isotherms almost corresponds to the path of the Kuroshio particularly in the area east of Japan where the Kuroshio flows eastward after leaving the coast. In many instances, these isotherms show meanders of wave lengths of a few hundred to more than a thousand nautical miles with amplitudes up to 300 miles (Ichiye, 1957). In most cases, the wave pattern of meanders seems to move eastward in the direction of the mean flow although the speed seems to be much less than the mean current. Also, the meanders of the isotherms usually showed a feature of tilting troughs and ridges in the direction from southwest to northeast. The harmonic analysis in space of the 15°C isotherm indicates that a large amplitude of meanders with a wave length of 1000 n. miles breaks into waves of higher harmonics after about one month or three consecutive charts. These features suggest that the meanders of the surface isotherms are quite similar to meanders of the westerlies in the upper atmosphere indicated by daily pressure charts. Rossby (1939) introduced an idea of planetary waves to explain the dynamics of westerlies meandering and many meteorological works were devoted to development of his theory since then. Ichiye (1950, 1956, 1957) tried to explain various features of the meanders of isotherms

from the planetary (Rossby) wave theories, for instance, the breaking of meanders into higher harmonics from the dispersion theory of Yeh (1949) and the tilting of troughs from Machta's model (1949). However, the original model of Rossby waves seems to be inadequate for the meanders of the western boundary currents even when they become zonal flows after leaving the continent, because the effect of variation of relative vorticity within the current or branch of multiple currents is much larger than β-effect. Therefore, in most cases meanders of the Gulf Stream may be treated as perturbations of a shear flow in a system uniformly rotating around a vertical axis, as discussed in the preceding sections.

On the other hand, when we consider the Gulf Stream Extension as a portion of the general circulation of the North Atlantic, the β-effect cannot be neglected. Fofonoff (1961) showed that an eastward mean flow is accompanied with wavy perturbations due to stationary Rossby waves, but for a westward flow the perturbations are not of an oscillatory type. He suggested that in a model of oceanic circulation of a rectangular basin, a basic flow in its southern half is almost zonal and westward, while a basic eastward flow in its northern half is superposed with stationary Rossby waves. Moore (1963) discussed the stationary Rossby waves in the interior part of the ocean including the effect of eddy viscosity. Both authors treated the barotropic model.

First, Fofonoff's (1961) theory is discussed. In a homogeneous ocean of a constant depth, components of two-dimensional stationary motion can be defined by a stream function ψ such that

$$u = \partial \psi / \partial y, \quad u = - \partial \psi / \partial x \tag{1 a, b}$$

Throughout this section, the x-axis is taken eastward and the y-axis northward. The equation of absolute vorticity

$$Z = \zeta + f = f - \nabla^2 \psi \tag{2}$$

is given by

$$u \partial Z / \partial x + v \partial Z / \partial y = 0 \tag{3}$$

where ζ is the relative vorticity. Substitution of (1) into (3) leads to

$$\partial \psi / \partial y \cdot \partial Z / \partial x - \partial \psi / \partial x \cdot \partial Z / \partial y = 0 \tag{4}$$

Therefore, Z becomes a function of ψ. In the simple case of intertial flows, Z is assumed to be a linear function of ψ. Further f is assumed to be $f_o + \beta y$. Then, we have

$$f_o + \rho y - \nabla^2 \psi = C_o + C_1 \psi \tag{5}$$

where C_o and C_1 are constants. The constant C_1 is determined from a condition that the velocity is zonal and equal to

$$\partial \psi / \partial y = U = \rho / C_1 \tag{6}$$

wherever ζ vanishes. The constant C_o is taken equal to f_o. Then Equation (5) becomes

$$(U/\beta) \nabla^2 \psi + \psi = U y \tag{7}$$

The ocean is assumed to be bounded by $x = \pm L_1$ and $y = 0, L$. The boundary condition is that $\psi = 0$ along the boundary. Introducing non-dimensional quantities

$$x' = \pi x / L, \quad y' = \pi y / L,$$

$$\psi' = \psi / |U| L, \quad k_o = (1/\pi) (\rho L^2 / |U|)^{1/2} >> 1 \tag{8}$$

and dropping primes, we obtain the non-dimensional form of (7)

$$k_o^{-2} \nabla^2 \psi \pm \psi = y \tag{9}$$

where the plus sign is associated with an eastward flow (positive U) and the minus sign with a westward flow. The boundary condition is that $\psi = 0$ at $y = 0, \; \pi$ and at $x = \pm a = \pm \pi L_1 / L$.

It is instructive to discuss the solution of (9) in relation to the Rossby waves. The homogeneous solutions of Equation (9) are

$$\psi_o = A_o \sinh k_o y, \quad \psi_n = A_n \cosh k_n x \sin n y \qquad \text{(10 a, b}$$

for the westward flow, where n is integers and

$$k_n^2 = k_o^2 + n^2 \qquad (11)$$

with $n \geqslant 1$. The solution of (9) satisfying the boundary condition can be obtained as follows: A_o is determined so that $\psi_o + \psi_p$ vanish at $y = 0$ and π, where $\psi_p = - y$ is the particular solution. A_n is determined by a Fourier expansion of $\psi_o + \psi_p$ at $x = a$. The complete solution is given by

$$\psi = - y + \pi \sinh k_o y / \sinh k_o \pi$$

$$+ 2 \sum_{n=1}^{\infty} (-1)^{n+1} k_o^2 \cosh k_n x \sin ny \left[n (k_o^2 - n^2) \cosh k_n a \right]^{-1} \qquad (12)$$

The solution for an eastward flow (positive sign) is

$$\psi = y - \pi \sin k_o y / \sin k_o \pi$$

$$+ 2 \sum_{n=1}^{\infty} (-1)^{n+1} k_o^2 \cos k_n x \sin ny \left[n (k_o^2 - n^2) \cos k_n a \right]^{-1} \qquad (13)$$

where

$$k_n^2 = k_o^2 - n^2 \qquad (14)$$

Equation (12) becomes simpler, because k_o is large compared with unity for the reasonable set of constants. For instance, $k_o \simeq 15.9$ for $L = 2500$ km, $\beta = 2 \times 10^{-13}$ (cm sec)$^{-1}$ and $U = - 5$ cm/sec. Therefore, $\cosh k_n x \simeq \cosh ko\, x$ for $n < k_o$, and the summation term of Equation (12) is replaced by

$(\psi_o + \psi_p) x \ \cosh k_o x/\cosh k_o a$. Further approximation for hyperbolic functions leads to

$$\psi \approx -\left[y - \pi \exp\left\{ - k_o (\pi - y)\right\}\right]\left[1 - \exp\left\{- k_o(a-x)\right\} - \exp\left\{ - k_o (a + x)\right\}\right] \tag{15}$$

This solution gives the circulation pattern, which has a narrow eastward jet with a width of $L/\pi k_o$ ($\approx$ 50 km) along the northern boundary and a broad westward current in the whole interior area. (Fofonoff, 1954). Solution (13) for positive U suggests that the streamlines consist of a complex series of eddies and that no eastward jet is evident. This solution can be interpreted as superposition of stationary Rossby waves on the mean westward flow.

A brief discussion of the barotropic Rossby waves will be presented here. The linearized two-dimensional momentum and continuity equations of homogeneous water with a uniform zonal current U (x- direction) are respectively given by

$$(\partial / \partial t + U \partial / \partial x) \ \vec{v} - f \vec{k} \times \vec{v} = - g \nabla \eta \tag{16}$$

$$(\partial/\partial t + U \partial / \partial x) \ \eta + H \operatorname{div} \vec{v} = 0 \tag{17}$$

where $\vec{v}$ and $\vec{k}$ are velocity vector and unit vector of the vertical direction; η is the surface elevation and H is the mean depth assumed as constant. A dimensionless notation (primed) is introduced in Equations (16) and (17) in terms of characteristic parameters:

$$(x, y) = (x', y') \ L, \quad \eta = \eta' H$$

$$(u, v,) = (u', v') \ U, \quad t = (L/U) t'$$

$$\beta = \partial f / \partial y = \beta' \ (L/ f) \tag{18}$$

When a prime is dropped, Equations (16) and (17) can be written as

$$R\,\delta \vec{v}/\delta t - (1 + \beta y)\,\vec{k} \times \vec{v} = -G\,\nabla\eta \tag{19}$$

$$\delta\eta/\delta t + \mathrm{div}\,\vec{v} = 0 \tag{20}$$

where

$$R = U/fL, \quad G = gH/(fUL) \tag{21a, b}$$

$$\delta/\delta t = \partial/\partial t \pm \partial/\partial x \tag{22}$$

in which a positive sign corresponds to an eastward flow and a negative sign a westward flow. The numerical constants used in the inertial circulation ($L = 2500$ km, $f = 10^{-4}\ \mathrm{sec}^{-1}$, $U = 5$ cm/sec, $\rho = 2 \times 10^{-13}\ (\mathrm{cm})^{-1}$) with $H = 4000$ m give

$$R = 2 \times 10^{-4}, \quad G = 3 \times 10^{3}, \quad \rho = 0.5 \tag{23}$$

The quasi-geostrophic relation for $\vec{v}$ to the order of R is given by

$$(1 + \rho y)\,\vec{v}/G = \vec{k} \times \nabla\eta - R\,(1 + \rho y)^{-1}\,(\delta/\delta t)\,\nabla\eta \tag{24}$$

Substituting (23) into (19) we have

$$R\,(\delta/\delta t)\,(\nabla^2\eta - 2\,\beta\eta_y - \eta/RG) + \beta\eta_x = 0 \tag{25}$$

in which the term $1 + \rho y$ is replaced by 1. When η is assumed to be of a form $\exp i(\sigma t + kx + \ell y)$, Equation (24) yields the frequency equation

$$R\omega\,(k^2 + \ell^2 + 2\beta\ell i + M) - \beta k = 0 \tag{26}$$

where

$$M = (RG)^{-1} \quad \omega = \sigma \pm k \tag{27 a, b}$$

In Equation (27b) a plus sign corresponds to an eastward flow and a minus sign a westward flow. The waves which satisfy the frequency Equation (26) are called Rossby waves. Equation (26) indicates that if the waves are stable, the amplitude should be changed exponentially unless the mode is purely zonal ($\ell = 0$). When k is assumed real and $\ell = \nu - i\kappa$, Equation (26) yields

$$R\omega\,(k^2 + \nu^2 + \rho^2 + M) - \rho k = 0. \quad \kappa = \beta \tag{28 a, b}$$

The waves become stationary when $\sigma = 0$. However, the waves with a constant amplitude are possible only for the eastward flow (positive sign in (27 b). In this case (28a) becomes

$$k^2 + \nu^2 \simeq \beta / R \tag{29}$$

by neglecting terms of the smaller order. Equation (29) is equivalent to the characteristic Equation (14), since βR^{-1} is equal to $\beta L^2 / U \; (= \pi^2 k_o^{\;2})$. On the other hand, when $\sigma = 0$ for the westward flow, Equation (28a) is satisfied only when k is imaginary. If we put $k = i k'$, (27a) becomes

$$\nu^2 + \beta / R \simeq (k')^2 \tag{30}$$

which is equivalent to (11). It is easily seen that in this case the eigen functions have an exponential function of x as a factor.

Moore (1963) treated a model of stationary wind-driven ocean circulation in a two-dimentional homogeneous system including the inertia and eddy viscosity terms. The velocity can be expressed by relations (1a, b) using a stream function ψ under an assumption of non-divergent flow. Since

$$\partial (f\psi) / \partial y = f u + \beta\psi \tag{31}$$

the Navier-Stokes equations in a β-plane can be written as

$$u \partial u / \partial x + v \partial u / \partial y = - \rho^{-1} \partial p' / \partial x + W + K \nabla^2 u \tag{32}$$

$$u \partial v / \partial x + v \partial v / \partial y - \beta\psi = - \rho^{-1} \partial p' / \partial y + K \nabla^2 v \tag{33}$$

by introducing a modified pressure p' defined by

$$p' = p + \rho f \psi \tag{34}$$

In Equation (32) W is a zonal wind stressfunction expressed as a body force. Cross differentiation of (32) and (33) yields the vorticity equation

$$\begin{aligned} &\epsilon (\partial \psi / \partial y) \, \partial (\nabla^2 \psi) / \partial x - \epsilon (\partial \psi / \partial x) \, \partial (\nabla^2 \psi) / \partial y + \partial \psi / \partial x \\ = \; &d W / d y + \delta \nabla^4 \psi \end{aligned} \tag{35}$$

after scaling the variables by use of (8) and dropping a prime. In this equation, new parameters

$$\epsilon = U/(\beta L^2) \qquad \delta = K/(\beta L^3) \tag{36 a, b}$$

are introduced (ϵ/δ is the Reynolds' number in a conventional sense) and the non-dimensional wind function W (with a prime) is equal to $W/(L\beta U)$. The wind stress torque dW/dy has a large negative value in an area between 20° N and 35° N in which the ocean circulation has a rather meridional structure. In the north and south of this area, the current shows a strong zonal structure and the wind stress torque is small. Therefore, we can neglect this term for the flow to be discussed. This assumption is contradictory to a model of wind-driven circulation treated by Carrier and Robinson (1962) whose solution shows a strong and narrow inertial jet at the latitude of maximum wind stress torque dividing anti-cyclonic gyre to the south and cyclonic gyre to the north. However, the problem to be discussed here is stability of a zonal current. Further, it is assumed that the zonal flow has a feature of a boundary layer current, that is,

$$\psi = 0, \quad \partial^2\psi/\partial y^2 = \partial u/\partial y = 0 \quad \text{on} \quad y = 0 \tag{37 a, b}$$

and on $y = y_1$ (< 1) ψ becomes the solution of (35) in the interior ocean in which the inertia and viscosity terms may be neglected, but the wind stress torque is finite. Boundary conditions (37 a, b) indicate that the flow is purely zonal on $y = 0$ and neither mass nor momentum is transferred across $y = 0$. (This boundary is not necessarily a solid barrier but may be taken as the center of the jet). From conditions (37 a, b), the stream function is assumed to be of a form

$$\psi = y F(x) + y^3 F_3(x) + \dots \tag{38}$$

Substituting (38) into (35) and equating the coefficients of y to zero, we have

$$\epsilon (F F''' - F' F'') + F' = \delta F'''' \qquad (39)$$

where a prime denotes differentiation with x. Once integrating (39) with x, we have

$$\epsilon \left[F F'' - (F')^2 \right] + F = - C + \delta F''' \qquad (40)$$

where C is an integration constant. The solution of (40) is given by

$$F = - C + C \exp (- m x) \qquad (41)$$

provided that

$$m^3 - \delta \epsilon C m^2 + \delta = 0 \qquad (42)$$

When $\delta << 1$, Equation (42) is reduced to

$$\epsilon C m^2 - 1 = 0 \qquad (43)$$

Equation (43) is the same as Fofonoff's (1954) characteristic equation for an inertial circulation, equivalent to (11) and (14). When $C > 0$, m is real and Equation (41) indicates a westward flow which vanishes at $x = 0$, decreasing uniformly from further east ($x = \infty$) if m is taken positive. When $C < 0$, a mean flow is eastward but m is imaginary and the flow is oscillatory. General Equation (42) has real roots only if

$$4 \delta^{-2} (\epsilon C)^3 > 27 \qquad (44)$$

whilst, in the contrary case, it has a pair of conjugate complex roots. It is easily seen that the westward flow ($C > 0$) always has complex roots. Since we may take $|C|$ as unity, relation (44) becomes

$$4 U^3 > 27 \beta K^2 \qquad (45)$$

for the westward flow. It is noted that the scale distance L is not included in this criterion. For $U = 10$ cm / sec, this inequality is valid if $K < 2.7 \times 10^7$ cm^2/sec. A simple algebraic calculation shows that the roots of (42) consist of one negative and two positive values if (45) is valid and one negative and

complex conjugate roots with a positive real part. However, the negative root for an eastward current is physically possible only when the number δ becomes of the order of unity or more or the eddy viscosity is sufficiently large, because otherwise, the absolute value of the negative root is so large that the velocity becomes unreasonably large, as x decreases from zero. For instance, for $L = 100$ km and $U = 10$ cm/sec, $\delta = 0.05$ and 0.5 respectively for $K = 10^7$ and 10^8 cm^2/sec, and the negative root for an eastward flow is -6.1 and -2.4, respectively. On the other hand, the negative root for a westward flow is -2.0 and -1.7 for the respective value of K. Therefore, for the westward flow the stationary current decreasing from east to west is possible for a relatively low eddy viscosity.

Moore (1963) further solved Equation (35) for the wind stress function $W = -\cos \pi y / L'$. He linearized the equation in a way similar to the boundary layer theory, assuming that the main interior current flows in the zonal direction, thus obtaining

$$- \epsilon U(y) \psi''' + \psi' = dW/dy + \delta \psi'''' \tag{46}$$

where $U(y)$ is a non-dimensional zonal velocity and is parameterized though it depends on y. For the realistic constants $U_o = 10$ cm/sec, $\beta = 10^{-1}$ (cm sec)$^{-1}$, $K = 2 \times 10^7$ cm^2/sec and $L = 5000$ km, corresponding to $\delta^{-1} \epsilon = 2.5 \times 10^2$, he obtained the streamlines showing damped Rossby waves in the western part of the northern half of the ocean, suggesting that the observed meanders of the Gulf Stream rather may be a feature of the stable solution than due to instability.

Il'in and Kamenkovich (1963) discussed Equation (35) by setting

$$\delta = e \epsilon^{3/2} \tag{47}$$

where e is a positive constant, since in general, $\epsilon \gg \delta$. Further, if the wind stress W is a linear function of y, it is assumed that

$$dW/dy = b; \quad \psi = W(y)\,F(x) \tag{48}$$

Then, Equation (35) yields

$$-e\,\epsilon^{3/2} F'''' + \epsilon\, b\,(F'F'' - FF''') + F' + 1 = 0 \tag{49}$$

instead of Equation (40). When an ocean is bounded by $x = 0$ and $x = 1$, the boundary conditions at these points are

$$F = F' = 0 \tag{50}$$

By use of boundary layer techniques, $F(x)$ is expanded into an asymptotic series by powers of ϵ like

$$F(x) = F_o(x) + \sqrt{\epsilon}\, F_1(x) + \epsilon\, F_2(x) + \dots$$

$$+ E_o(q) + \sqrt{\epsilon}\, E_1(q) + \dots + G_o(r) + \sqrt{\epsilon}\, G_1(r) + \dots \tag{51}$$

where

$$q = x/\sqrt{\epsilon}, \qquad r = (1 - x)/\sqrt{\epsilon} \tag{52 a, b}$$

and E_n or G_n is significant only near $x = 0$ or $x = 1$, respectively. Substituting (51) into (49) and equating coefficients of the same power of ϵ, we have

$$F_o' + 1 = 0 \dots \tag{53}$$

$$-e\,E_o'''' + b\left[E_o'\,E_o'' - (F_o(o) + E_o)\,E_o'''\right] + E_o = 0 \tag{54}$$

$$-e\,G_o'''' - b\left[G_o'\,G_o'' - (F_o(1) + G_o)\,G_o'''\right] - G_o' = 0 \tag{55}$$

where a prime indicates differentiation with x, q and r, respectively. The boundary conditions become

$$E_n(o) + F_n(o) = 0, \quad E_n'(o) = 0, \quad E_{n+1}'(o) + F_n'(o) = 0 \tag{56}$$

$$G_n(o) + F_n(1) = 0, \quad G_o'(o) = 0, \quad -G_{n+1}'(o) + F_n'(1) = 0 \tag{57}$$

In expansion (51), $E_n(q)$ and $G_n(r)$ must decrease exponentially as q and r become infinite. Therefore, in the range of large q and r, non-linear terms of (54) and (55) are neglected and we have

$$-e\,E_o''' - b\,F_o(o)\,E_o'' + E_o = 0 \qquad q \gg 1 \tag{58}$$

$$-e\,G_o''' + b\,F_o(1)\,G_o'' - G_o = 0 \qquad r \gg 1 \tag{59}$$

From Equation (53), $F_o(x) = 1 - x$. Then $G_o \approx 0$, owing to the conditions that $G_o(o) = G_o'(o) = 0$.

When we assume that $G_o(q)$ is of a form $\exp \chi q$, χ is two roots of the cubic equation

$$e\chi^3 + b\chi^2 - 1 = 0 \tag{60}$$

with $\mathrm{Re}\,\chi < 0$. When $b > 0$ (corresponding to the southern half of the northern ocean), we have

$$\chi_1 = -1/\sqrt{b} + O(e), \quad \chi_2 = -b/e + O(e) \tag{61 a, b}$$

for small values of e. These roots indicate that the boundary layer has two kinds with thickness of the order of $\sqrt{\epsilon}$ and $e\sqrt{\epsilon}$, respectively. When $b < 0$ (northern half), we have

$$\chi_1 = i/\sqrt{|b|} - e/2b^2 + O(e^2), \quad \chi_2 = -i/\sqrt{|b|} - e/2b^2 + O(e^2) \tag{62}$$

for small values of e. The boundary layer is larger than in the former case and of the order of $\sqrt{\epsilon}/e$, but the functions show damped oscillation with a variable q. This corresponds to the Rossby waves in an eastward current, as discussed by Fofonoff (1961) and Moore (1963) for a pure inertial flow. It is easily seen from Equation (49) that this model becomes Carrier and Robinso (1962) model for small values of e and Munk's (1950) model for large values

of e. Therefore, Equation (54) is essentially the same as those treated by Moore (1963).

Welander (1961) treated baroclinic Rossby waves propagating in a zonal ocean current decreasing exponentially with depth. He used the x-y-p system with the vertical velocity $\omega = \partial p / \partial t$ and φ (=g z) the geopotential of a pressure surface. It is assumed that the perturbation is geostrophic although the basic current is zonal and that a hydrostatic balance exists in the vertical direction. Then, equations of motion in the horizontal and vertical directions are, respectively.

$$-fv = -\partial\varphi / \partial x, \quad fu = -\partial\varphi / \partial y \tag{63}$$

$$\alpha = -\partial\varphi / \partial x \tag{64}$$

where α is the specific volume ($\alpha = \rho^{-1}$). Equations of continuity and conservation of mass are given by

$$\partial u / \partial x + \partial v / \partial y + \partial \omega / \partial p = 0 \tag{65}$$

$$\partial \alpha / \partial t + U \partial \alpha / \partial x + f U_p v + A_p \omega = 0 \tag{66}$$

All the dependent variables in (63) to (66) are for the perturbation except U and A, which are respectively, the zonal velocity and specific volume of the basic system, satisfying the relations

$$fU = -\Phi_y, \quad A = -\Phi_p \tag{67}$$

Derivatives of the basic quantities are denoted by a subscript. Equation (66) can be derived as the perturbation of the total equation:

$$(\partial / \partial t + u\partial / \partial x + v\partial / \partial y + \omega\, \partial / \partial p)\, \alpha = 0 \tag{68}$$

in which dependent variables are the resultant of the basic quantities and their perturbations. Solving ω from (66) by use of (63a) and (64), we have

$$\omega = A_p^{-1} (\partial^2\varphi / \partial p \partial t + U \partial^2\phi / \partial x \partial p - U_p \partial\phi / \partial x) \qquad (69)$$

Substitution of velocities (63 a, b) into (65) yields

$$\partial \omega / \partial p - \beta f^{-2} \partial \phi / \partial x = 0 \qquad (70)$$

retaining the β-effect. Since ω is expressed in terms of function ϕ with (69), (70) represents the perturbation equation for φ. The boundary condition is given by

$$\omega = 0 \text{ at } p = 0; \qquad \phi = 0 \text{ at } p = \infty \qquad (71\text{ a,}$$

where the condition at $p = 0$ is expressed in terms of ϕ by use of (69).

Introducing a wave form

$$\varphi = \varphi^* (y, p) \exp ik (x - ct) \qquad (72)$$

Equation (70) and boundary conditions (71 a, b) transform into

$$(U - c) \partial (A_p^{-1} \partial\varphi^* / \partial p) - \left[(U_p A_p^{-1})_p + \beta f^{-2} \right] \varphi^* = 0 \qquad (73)$$

$$(U - c) \partial\phi^* / \partial p = U_p \varphi^* \text{ at } p = 0; \; \varphi^* = 0 \text{ at } p = \infty \qquad (74)$$

Two features can be seen immediately from (73) and (74 a, b). Since k does not occur in these equations, the wave speed is independent of k. Also, the dependence of U and A on y can be treated parametrically, since no y-derivatives appear.

The basic quantities are assumed to be

$$U = U_o \exp (-p / p_o), \quad A = A_o + A_1 \exp (-p/p_o) \qquad (75\text{ a, b})$$

where p_o is the scale pressure. Introducing non-dimensional parameters

$$P = p/p_o, \quad c^* = c / U_o, \quad \tau = \beta A_1 p_o / (f^2 U_o) \qquad (76\text{ a, b, c})$$

(73) and (74 a, b) change to

$$\left[\exp (-P) - c^* \right] d (\exp P \, d\varphi^* / dP) / dP + \tau \varphi^* = 0 \qquad (77)$$

$$(1-c^*)\,d\phi^*/dP + \phi^* = 0 \text{ at } P = 0; \quad \phi^* = 0 \text{ at } P = \infty \qquad (78\text{ a, b})$$

When $c^* = 0$ (stationary waves), the general solution has a form $\exp \kappa P$ from which the frequency equation is given by

$$\kappa(\kappa + 1) + \tau = 0 \qquad (79)$$

When $\tau < 0$ corresponding to negative U_o (westward flow), one root of (79) is positive and cannot satisfy (78) and the elementary solution for another root cannot satisfy (78 a). Therefore, no stationary waves occur. If $\tau > 0$, both roots always have negative real parts and the stationary waves are possible. For $\tau > 1/4$, the vertical structure is of exponential type; for $\tau > 1/4$ of oscillatory type. In the real ocean, τ is small and the former type is usually possible.

In a general case of $c^* \neq 0$, the new variable

$$s = \exp(-P)/c^* \qquad (80)$$

transforms Equation (77) and (78 a, b) into

$$s(s-1)\,d^2\phi^*/ds^2 + \tau\phi^* = 0 \qquad (81)$$

$$(1-c^*)\,d\phi^*/ds - c^*\varphi = 0 \text{ at } s = 1/c^*; \quad \phi^* = 0 \text{ at } s = 0 \qquad (82\text{ a, b})$$

The solution satisfying (82 b) is expressed in terms of the Legendre function which diverges at $s = 1$, except when $\tau = -n(n+1)$ $(n=1, 2, 3..)$ and the solution becomes a polynomial. In general, therefore, $c^* > 1$ in order to have a finite solution. Only solutions for $\tau < -2$ i.e. for a westward current are found. At middle latitudes, the physical constants lead to $\tau \approx 0.1$. Therefore, there are no traveling waves in a westward current. In the polynomial case (for an eastward flow) waves remain stationary relative to the surface current. Therefore, as in a barotropic model discussed

before, the Rossby waves of stable nature cannot propagate in the westward flow in a baroclinic model with the density varying exponentially with depth. It will be worthwhile to develop a similar theory in a two-layer system. Also, it is necessary to check the effect of horizontal shear of the basic current U(y) by including advective terms in Equations (63 a, b).

Ichiye (1958) discussed a problem of long waves including Rossby waves generated by the zonally-moving wind stress of a limited meridional extent in a two-layer ocean. There is no basic current in his model. His result showed that the waves become resonant (unstable) for a disturbance propagating westward in a range of Rossby waves while there is no resonance for an eastward disturbance. It is yet to be studied the response of an eastward or westward flow to a moving disturbance of a pressure system or a wind system.

7. Application of a principle of Constant Absolute Vorticity Trajectories.

All the theories discussed in the previous sections deal with the meanders as a perturbation of the basic current and thus are based on the assumption that the amplitude of meanders are small enough to neglect its square or higher powers. However, the observations indicated that meanders of the Gulf Stream and the Kuroshio seemed to be, on many occasions, much larger than assumed in small amplitude theories. Rossby (1940) proposed a theory on meanders of a narrow planetary jet from conservation of the absolute vorticity. His theory was applied to forecasting change of the stratospheric jet stream in meteorology and was known as the CAV method. Ichiye (1955b) applied this method to courses of the Kuroshio to the east of 142° E in five surveys of different seasons in 1953 to 1954. He used the velocity distributions determined with dynamic calculations based on the 800-decibar level to compute the volume and momentum transport of the Kuroshio. His results showed that the paths of the current were within the first half wave length (300 to 500 km) in agreement with predicted ones based on observed quantities at the initial point where a path was straight. The deviation of the actual paths from theoretical ones after one half wave length was explained as an effect of vorticity transport due to lateral mixing process (Ichiye, 1957). Defant (1956) also noticed similarity between the inertia motions in the equatorial zone studied by Whipple (1917) and Rossby's meandering trajectories of a planetary jet. He speculated that the eastward flow, such as the Gulf Stream extension,

may meander with large amplitudes, occasionally discharging eddies and yet remain narrow owing to its stability, as shown by Rossby's trajectories, while the westward flow like the North Equatorial Current is essentially unstable as eastward trajectories indicate and thus, the disturbances are widely spread so as to make the current broad by lateral transfer of momentum. Saint Guily (1957) modified Rossby's original theory by including effects of variable velocity and volume transport along the path to explain schematically excessive meanders of the Gulf Stream as observed during Operation Cabot.

The equation of a trajectory of constant absolute vorticity is derived for a two-layer system, as follows. It is assumed that only the upper layer with a depth D is in motion and the lower layer is infinitely deep. Then, the potential absolute vorticity is constant along a streamline (Charney, 1955) or

$$(\zeta + f)\, D^{-1} = \text{const. (on a streamline)} \qquad (1)$$

where ζ is the relative vorticity. The volume transport is constant along a strip in the upper layer bounded by two neighboring streamlines and the interface. Therefore, we have

$$\bar{q}\, B\, \bar{D} = \text{const} \qquad (2)$$

where q is the velocity and E is the width between the streamlines and a bar indicates the average over a cross-section of the strip.

The relative vorticity consists of curvature and shearing terms, like

$$\zeta = qK + \partial q / \partial n \qquad (3)$$

where K is the streamline curvature and n is a coordinate normal to the

streamline. When two streamlines of the strip are close enough, the lateral change of q and k can be neglected and the average of ζ can over a cross-section of the strip or jet can be expressed by

$$\zeta \approx \overline{qK} \approx \overline{q} \cdot \overline{K} \tag{4}$$

In the following, a bar denoting the average is dropped. Then after being averaged over a cross-section, Equation (1) becomes

$$D^{-1} (f + q \cdot K) = \text{const. (on a streamline)} \tag{5}$$

If the origin of x-y coordinates is taken at a point of a straight portion of the flow and x- and y- axes are taken in the east and north direction respectively, Equation (5) yields

$$\beta y + K (V / DB) = f_o (D / D_o - 1) \tag{6}$$

where the suffix o denotes the value at $y = 0$ and V is equal to $(D B q)_o$ or the volume transport of a jet and constant along the jet from Equation (2). Saint Guily (1957) obtained Equation (6) from a normal component of equations of motion in a natural coordinate system.

$$K q^2 + f q = - \partial p / \partial n \tag{7}$$

If Equation (7) is integrated between two sides of the jet, the righthand side gives pressure difference between both sides and is constant along the jet because the sides consist of isobars. When Equation (2) is used, this integrated equation becomes (5).

When $y(x)$ denotes the coordinate of the path of the jet, curvature $K(x)$ can be expressed by

$$K(x) = y'' \left[1 + (y')^2 \right]^{-3/2} \tag{8}$$

where a prime indicates differentiation with x. If the velocity q is constant, D is also constant along the jet and Equation (6) becomes

$$m y'' + y \left[1 + (y')^2 \right]^{-3/2} \qquad (9)$$

in which

$$m = q_o / \beta \qquad (10)$$

Equation (9) can be solved analytically with the initial conditions that

$$y(o) = 0, \quad y'(o) = \tan \theta \qquad (11\ a, b)$$

The first integration of (9) yields

$$y' = \left[(\cos \theta + y^2/2m)^{-2} - 1 \right]^{1/2} \qquad (12)$$

It is at once seen that the ordinate has an extreme

$$y_o = 2m(1 - \cos \theta)^{1/2} \qquad (13)$$

which is the amplitude when the path is oscillatory. Introducing a new variable ϕ defined by

$$k^2 \sin^2 \phi = (y^2/2m)(1 + \cos \phi + y^2/2m)^{-1} \qquad (14)$$

with

$$k^2 = (1 - \cos \theta)/2 \qquad (15)$$

we can integrate (12) which becomes

$$x = \sqrt{m} \cos \theta \int_o^\phi (1 + k^2 \sin^2 \phi / \cos \theta)(1 - k^2 \sin^2 \phi)^{3/2} \, d\phi \qquad (1$$

or

$$x = \sqrt{m} \left[2E(\phi, k) - F(\phi, k) - k^2 \sin \phi \, (1 - k^2 \sin^2 \phi)^{-1/2} \right] \qquad (17)$$

where $F(\phi, k)$ and $E(\phi, k)$ is Legendre's elliptic integral of the first and second kind, respectively. An extreme y_o corresponds to $\phi = \pi/2$ or the value of x_o defined by

$$x_o = \sqrt{m}\,(2\overline{E} - \overline{K}) \qquad (18)$$

where $\overline{E}$ and $\overline{K}$ are complete elliptic integrals, The paths for $\theta = 45°$ 90°, 117° 30', 180° are shown in Fig. 8. When θ reaches 117° 30', the path touches itself (occlusion) making looplike meanders. When θ exceeds 130° 44', the jet flows westward, making a series of loops and at $\theta = 180°$, the path forms only one loop at $x = -\infty$. The values of x_o (a quarter of a

wave length) y_0 (half amplitude) for different values of θ

θ (°)	15	45	75	90	117.5
$y_0/\sqrt{m}$	0.26	0.76	1.22	1.41	1.71
$x_0/\sqrt{m}$	1.55	1.39	1.07	0.85	0.32

where the values of m in km. are for 5, 25, and 100 cm/sec of q, respectively, 47, 105 and 210 at the Equator and 55, 120 and 240 at 45° latitude.

In order to discuss the effect of change of depth D on the course of meanders, it is assumed first that q varies with the distance ℓ along the jet from an origin for constant D and B. Then, Equation (6) becomes

$$K q(\ell) + \beta y = 0 \qquad (19)$$

When q decreases with ℓ, the curvature K increases, the angle of the path with y- axis θ increases and steepness (ratio of amplitude to wave length) of meanders increases, leading occasionally to occlusion. On the other hand, when q increases with ℓ, K decreases, θ decreases, the steepness decreases and the path of the jet is more straight. Next, it is assumed that the depth D is dependent on x only and that B is constant. Then, if y is replaced by $y_1 = y - (f_0/\beta)(D/D_0 - 1)$ in Equation (6), the same conclusion as above will be obtained, i.e. steepness of meanders increases when the depth increases with x and vice versa. The only difference is that the axis of the meanders (the mean path) is $y_1 = 0$ in this case instead of $y = 0$ (x- axis) for the case of constant q. When the depth is a linear function of y like $D = D_0(1 + sy)$, Equation (10)

yields

$$K q_o (1 + sy)^{-1} + (\beta - s f_o) y = 0 \qquad (20)$$

when $s = s_o = \beta f_o^{-1}$, the β-effect is compensated by the effect of depth and the jet flows straight, as noticed by Ekman (1923) and Neumann (1955). From Equation (20), a general rule can be obtained according to $s < s_o$ or $s > s_o$. When $s < s_o$, steepness of meanders is less for $s > 0$ and more for $s < 0$ than for a constant depth. When $s > s_o$ and $s > 0$, an eastward jet becomes unstable and an occlusion occurs immediately, while a westward jet shows stable meanders. When $|s| > s_o$ and $s < 0$, vice versa.

Ichiye (1957) extended Equation (6) to a case that the initial angle θ and the velocity q vary periodically with time and explained the coupling of a cyclonic detour to the south of the Kuroshio between 135° and 140° E and its northward anti-cyclonic meander east of 142° E from an equation modified from (6). Also, he considered the horizontal transport of relative vorticity across both sides of a jet through eddy viscosity, approximated it with a linear function of a distance ℓ and equated it with the right-hand side of (6). It is shown that the influx of positive vorticity causes less cyclonic curvature and farther northward extension of the current, suggesting that an occasional occlusion of the Kuroshio east of 150° E is due to lateral influx of relative vorticity.

Warren (1962) studied the effects of bottom topography on the course of the Gulf Stream north of Cape Hatteras. From conservation of absolute vorticity, he derived the equation which is essentially the same as Equation (6) excepting that D is equal to the actual depth instead of the

depth of the upper layer, He considered an example that D is a linear function of both x and y like

$$D = D_o + s_1 x + s_2 y \tag{21}$$

Further, instead of a simple form of velocity q and volume transport qDB, he used quantities integrated across the entire stream width, such as

$$V = \int q\,D\,dn, \quad M = \int q^2 D\,dn, \quad T = \int q\,dn \tag{22}$$

Then, the averaged vorticity equation similar to (9) becomes

$$K(x) + N_1 y - N_2 x = 0 \tag{23}$$

where

$$N_1 = (\beta V - f_o T s_2)\, M^{-1}, \quad N_2 = f_o\, T s_1\, M^{-1} \tag{as a, b}$$

By rotation of x-y and transformation of $(x$-$y)$ system into $(x'$-$y')$ system, Equation (23) is changed into the same form as Equation (9) except that

$$m^{-2} = N_1^{\,2} + N_2^{\,2} \tag{24}$$

instead of (9). The analytical solution of the transformed equation is given by relations (16) or (17) with x and y replaced by x' and y'. In order to determine the path of the Gulf Stream east of Cape Hatteras, Warren (1963) applied this equation to a series of small segments starting at a point of no curvature. He used the observed value of velocity and direction of the Gulf Stream at a starting point only and successively proceeded to each segment in which s_1 and s_2 are determined from a chart. One of his results which showed the best agreement with an observed path is shown in Fig. 8. On some occasions, observed paths showed larger meanders than calculated, suggesting that an eddy influx of vorticity across the sides may be important as in the case of the Kuroshio, discussed by Ichiye (1957).

Greenspan (1963) also noticed that the 10-, 100- and 1000-fathom lines almost coelesce just south of Cape Hatteras and suggested that the Gulf Stream being stable as an inertial boundary current south of the Cape becomes unstable owing to a sudden increase of the depth north of it. His argument was based on his criterion for the existence of inertial boundary layers (Greenspan, 1962). In a homogeneous ocean with a variable depth D or in a two layer system with an upper layer of a thickness D and with a motionless lower layer, conservation of volume and absolute vorticity yields two equations (Charney, 1955).

$$uD = -\partial\psi/\partial y, \quad vD = \partial\psi/\partial x \tag{25}$$

$$(f+\zeta)/D = F(\psi) \tag{26}$$

where F is an arbitrary function and ζ is the relative vorticity. As in Section 6, these equations can be expressed with non-dimensional variables by introducing U, L, H and f_o as characteristic velocity, length depth and Coriolis' coefficient, respectively. Then, the absolute vorticity is given by

$$f + \epsilon \nabla \cdot (D^{-1} \nabla \psi) \tag{27}$$

where

$$\epsilon = U/(2 f_o L) \tag{28}$$

is a small dimensionless parameter if L is taken as the lateral dimension of the ocean. In Equation (27) and the following, all the quantities are expressed in a dimensionless form. A new coordinate system $\xi = \xi(x, y)$, $\eta(x, y)$ is introduced by use of a conformal transformation

$$\xi + i\eta = w(x + iy) \tag{29}$$

with w an analytic function. Furthermore, let $\xi = \xi_B$, $\xi < \xi_B$ and η denote the coast, the ocean and a coordinate normal to the coast, respectively.

The stream function ψ varies rapidly across the narrow width of a coastal boundary layer from its interior value $\Psi(\xi, \eta)$ to zero at the coast. Therefore, by introducing a stretched boundary coordinate

$$z = (\xi - \xi_B)\, \epsilon^{-a} \qquad (a > 0) \tag{30}$$

Thestream function may be divided into two parts:

$$\psi = \phi(z, \eta) + \Psi(\xi, \eta) \tag{31}$$

Transforming Equation (26) $\Big($$\zeta$ substituted by (27) by use of (29) and (30)$\Big]$ and equating the coefficients of $\epsilon^{-1/2}$ for $a = 1/2$ to zero, we have

$$\phi_{zz}(J D)^{-2} + f D^{-1} = \Omega(\Psi + \phi) \tag{32}$$

where

$$J^2 = x_\xi{}^2 + y_\xi{}^2 \tag{33}$$

and Ω is an arbitrary function. If an inertial boundary layer is to exist, then ϕ must vanish at the seaside edge of the boundary layer $(z = -\infty)$. Therefore,

$$\Omega(\Psi) = f D^{-1} \tag{34}$$

with $\Psi = \Psi(\xi_B, \eta)$. Equation (32) becomes

$$\phi_{zz} = (J D)^2 \left[\Omega(\Psi + \phi) - \Omega(\Psi)\right] \tag{25}$$

For large z, ϕ is so small that the bracketed term can be expanded as a Taylor series. Thus,

$$\phi_{zz} \approx (J D)^2 (\partial \Omega / \partial \Psi)\, \phi \tag{36}$$

when $z \approx -\infty$. However, ϕ approaches asymptotically zero as z increases only if the coefficient of ϕ in (36) is positive (ϕ becomes oscillatory with z if the coefficient is negative). Therefore, the condition for existence of the boundary layer is simply

$$\partial Q / \partial \Psi > 0 \qquad (37)$$

for $\xi \approx \xi_B$. By use of (34), this condition becomes

$$\partial \Psi_{\eta} \, (f / D)_{\eta} > 0 \qquad (38)$$

However, Ψ is the stream function of the interior flow and thus $-\Psi_{\eta}$ is the ξ component of the interior flow in the direction of increasing η. If $\vec{v}_I$ and $\vec{n}$ denote the vectors of the interior flow velocity and the unit vector normal to the boundary in the outward direction, we have

$$\Psi_{\eta} = - \vec{v}_I \cdot \vec{n}.$$

Therefore, condition (38) can be written as

$$(\vec{v}_I \cdot \vec{n}) \; \partial (f / D) / \partial \eta < 0 \quad \text{at} \quad \xi = \xi_B \qquad (39)$$

Greespan (1963) concluded that the effect of topography south of Cape Hatteras controls the path of the Gulf Stream as an inertial boundary current since the criterion (39) is satisfied there owing to a slight change of D, while $\partial f / \partial \eta > 0$ and $\vec{v}_I \cdot \vec{n} < 0$. Just south of the Cape, the positive value of $\partial D / \partial \eta$ exceeds the effect of the northward increase of f and the criterion (39) is violated. He suggested that the control of topography may be effective even to a two-layer system through some intermediary means. Also, he speculated that the location and flow structure at the separation point just south of the Cape may change with the order of several weeks according to criterion (39), since in Warren's calculation (1962) the initial condition for the Gulf Stream somewhere just north of the Cape must be taken differently for each meander.

8. Energy Transfer Between the Meanders and the Mean Flow

Since vorticity transfer through the sides of the flow seems to play an important role in meanders, as discussed in Section 7, Ichiye(1957) tried to determine momentum transfer in the Kuroshio due to meanders. The G E K data obtained in the area between 135° E and 150° E in March and May, 1956 were divided in two groups east and west of longitude 141° E, each group containing data on seven to nine transects across the currents consisting of stations of less than ten miles apart along each transect. In each transect, the current at each station was decomposed into components parallel and normal to the mean flow over the transect. Then each component was grouped again according to the distances from the axis (having maximum speed) of the current, and averaged. Also, for each group, correlations of deviations of velocity u' and v' were computed, where u and v are velocity components normal and parallel to the mean flow determined for each transect. Therefore, if a bar denotes the average over a group consisting of stations of equal distance from the current axis, the energy transfer from meanders or (eddies) to the mean flow is given by

$$\rho \, \overline{u' v'} \; \partial \overline{v} / \partial x \tag{1}$$

where x denotes the distance normal to the mean flow. The computed values of this transfer and mean speed $\overline{u}$ for the western and eastern region are shown in Fig. 10-a. In the western region, the energy is transferred from the mean flow to meanders to the left side of the axis (facing downstream) and in the reverse direction to the right side but in the eastern region vice versa.

Webster (1961a) computed the energy transfer from eddies to the mean flow at two transects across the Gulf Stream from GEK data, using a relation similar to (1) except the bar denoting a time average of a quantity. One transect is between Miami and Gun Cay, Bimini. A total of 632 GEK readings from 42 cruises made from 1956 to 1958 were divided into eleven longitudinal zones and the average shear $\partial v / \partial x$ and Reynolds stress $u'v'$ are computed for each zone. Another transect off Onslow Bay was crossed almost normal to the Gulf Stream axis one hundred and twenty times during some thirty consecutive days. The GEK readings were divided into eleven zones and treated in the same ways as Florida Straits data. The results shown in Fig. 10b indicate that the energy is transferred from meanders to the mean flow, contrary to the conventional speculation that the meanders represent a mechanism for frictional dissipation of the mean flow. This feature is similar to the energy transfer from large scale eddies to the mean zonal flow in the atmosphere determined by Starr (1953), whose results are shown in Fig. 10c by plotting the production of zonal kinetic energy and the relative angular velocity as a function of latitudes. Though the energy transfer has a similar feature between two transects, its values averaged over the entire length of each transect are quite different and equal to 79×10^{-4} and 3×10^{-4} ergs cm^3/sec for off Onslow Bay and the Florida Straits, respectively. Since the kinetic energy of the mean surface flow averaged across the stream is 7.3×10^3 ergs/cm^3 and 8.6×10^3 for off Onslow Bay and the Florida Straits, respectively, the calculated rate of energy transfer doubled the mean surface kinetic energy in 11 in the former and 329 days in the latter. Webster also discussed

the energy equation for the mean flow in a steady state.

$$\vec{V} \cdot \nabla_h E + \bar{u} \nabla \cdot \rho \overline{u'\vec{v'}} + \bar{v} \nabla \cdot \rho \overline{v'\vec{v'}} + \vec{V} \cdot \nabla_h \bar{p} + \vec{V} \cdot \vec{F} = 0 \qquad (2)$$

which can be obtained from equations of motion in a horizontal plane and $\vec{V} = (\bar{u}, \bar{v})$, $\vec{v'} = (u', v', w')$, $\nabla_h = (\partial / \partial x, \partial / \partial y)$, $\nabla = (\partial / \partial x, \partial / \partial y, \partial / \partial z)$, $E = (\bar{u}^2 + \bar{v}^2)/2$ and $\vec{F}$ is the frictional force. Integrating (2) over a cross-section of the current, he estimated that the contribution to the kinetic energy of the mean flow by horizontal motion represented by the sum of the second and third term of (2) comes mainly from the term given by (1) while the effect of wind stress expressed by the last term of (2) is less than the contribution from (1) by one to two orders of magnitudes.

Webster (1961b) analyzed the data of the transects off Onslow Bay and found that the amplitudes of harmonic analysis of the location of the 20° isotherm at 200 m depth and of the velocity maximum determined by GEK along the transects have peaks at 4 and 7 days with values of 6 miles and 3.7 miles, respectively. He concluded that there is no definite evidence for meanders off Onslow Bay as relating to the tidal period, suggesting that they may be related to local meteorological disturbances. He proposed a model of meanders in this area consisting of those associated with an intense offshore current with one to four days' duration and those accompanying a broad onshore current of four to seven days' duration and of a wave length of a hundred kilometers in contrast with long meanders north of Cape Hatteras having wave lengths of 1000 km or more and periods of several months.

Oort (1963) computed co-variance in time of simultaneous cross current velocity (u') temperature (T') and density (ρ'), $\overline{u'T'}$ and $\overline{u'\rho'}$

from three transects including the one off Onslow Bay analyzed by Webster (1961a). Other transects are 70 crossings along the 30° N parallel off Jacksonville, in October-November, 1961 and 20 crossings along lines shifting slowly downstream off Cape Hatteras in October, 1962. He used surface data of temperature, density and GEK readings, as well as BT data for temperature at 30 m, 60 m and 100 m with surface GEK data for currents. It was found that the effect of a rotation of the coordinate axes by about 10° did not change the order of magnitude and sign of $u'T'$ suggesting that $\overline{v'T'}$ is amall compared with $\overline{u'T'}$. Also, since a slight rotation of axis may make the mean cross current velocity $\overline{u}$ vanish, the total heat transport across the stream $\overline{uT}$ can be well approximated by the eddy heat transport $\overline{u'T'}$. The heat transport $(\overline{u'T'})$, temperature and the downstream velocity at the surface at three transects are shown in Fig. 11. The results at three transects show that the offshore heat flux coincides approximately with the place of maximum current. This may explain the appearance of a warm core to the right of the current peak (facing downstream), which is apparent in the data shown in this figure. Also, these data indicate heat fluxes from the colder coastal waters to the warmer ocean in all three transects. The surface heat flux averaged along three transects is 1.8 ± 1.3, 3.5 ± 1.6 and 0.46 ± 0.55 (°C cm/sec), respectively off Cape Hatteras, Onslow Bay and Jacksonville. The average density flux $\overline{u'\rho'}$ is -0.9 ± 0.5, -1.8 ± 0.6 and -0.15 ± 1.9 (10^{-3} g sec^{-1} cm^{-2}) from north to south, again indicating the mass transport against the horizontal density gradient. The heat flux

at deeper layers shows the same tendency as the surface values and its magnitude averaged on a transect is three to ten times the surface value off Jacksonville. However, values of the subsurface heat flux is subject to more errors due to the use of surface currents instead of currents at corresponding depths, since a strong vertical shear usually exists near the surface. An idea of the order of magnitude of computed heat flux may be gained by calculating the eddy heat flux normal to a vertical wall along the Gulf Stream for a distance of 1000 km and with a depth of 100 m the heat transport $\rho C \overline{u'T'}$ across the wall (C, heat capacity of sea water) becomes 10^{18} cal day^{-1} corresponding to $\overline{u'T'} = 10$ (°C cm sec^{-1}). This value is comparable with the calculated annual poleward heat transport of 20×10^{18} cal day^{-1} at 30° N along a distance of 6700 km of the entire North Atlantic Ocean. Therefore, the meanders play an important role in maintenance of the Gulf Stream through thermal process as well as through momentum transport discussed by Webster.

Chew (1958) proposed a hypothesis that meanders due to internal waves of a semi-diurnal tidal period may cause fluctuations of cross stream components of surface currents measured with GEK between Miami and Cat Cay. Chew (1961) also determined the average surface cross stream component on this section from 500 GEK data over eight years and 1100 data from Pillsbury's measurements. The distribution indicates a westward (offshore) flow for a distance of 20 km from the Florida Coast with maximum speed of 10 cm/sec and an eastward (onshore) flow to the east until Cat Cay with the maximum speed of

14 cm/sec at about 79°40'E. He attributed the divergence near the coast to the lateral Ekman layer due to the coast and lateral eddy viscosity. It is yet to be seen how this systematic distribution of the mean cross current component may influence the momentum and heat transfer across the stream determined by Webster (1961a) and Oort (1963).

9. Model Experiments for Meanders

A model experiment of the wind-driven ocean circulation by use of a rotating circular tank was initiated by Von Arx (1952), who constructed a model of the entire northern hemisphere in a paraboloidal basin of two meters in diameter and of a focal length of 0.5 meter. His results showed a strong similarity to the actual circulation both in the North Atlantic and North Pacific Ocean to details of the meanders of the Gulf Stream between Cape Hatteras and the Grand Banks and of the cyclonic bend of the Kuroshio off the southern coast of Japan. The occurrence of a western boundary current like the Gulf Stream and its meandering is mainly due to the β-effect, as discussed by many authors and it was found that the similarity in the β-effect between the prototype and the model may be established by changing the depth D of a model according to the relationship

$$(\partial f/\partial y)_p = \left[\partial f/\partial y + (f + \zeta)\, D^{-1}\, \partial D/\partial y\right]_m \qquad (1)$$

where subscripts p and m refer to prototype and model, respectively; y is the poleward coordinate (or the radial coordinate in the model), ζ is the relative vorticity. Further, since for a flat basin $(\partial f/\partial y)_m = 0$ and also, ζ is usually negligible compared with f, Equation (1) indicates that the constant increase of depth D of the model towards the rotation center according to (1) may simulate the β-effect. A rotating model experiment with a 1.5 meter diameter basin made by Ichiye and Plutchak (1962)for the circulation in the Gulf of Mexico showed that the modification of depth according to (1) produced an anti-cyclonic loop of

the flow after entering the Gulf through the Yucatan Straits, as observed in hydrographic surveys.

The occurrence of meanders in the eastward jet was also noticed by a qualitative experiment in a pie-shaped basin, where an ocean circulation similar to the real one was produced by suitable distribution of a sink and a source (Stommel and others, 1958). Faller (1961) found that the compensation flow in the bottom Ekman layer combined with a radial depth variation in the model yields an equation of a stream function similar to the equation for a wind-driven ocean circulation. A slight modification of his analysis is summarized as follows:

For a fluid in a rotating model basin, r, θ and z are taken radially outward, in the direction of rotation (counter-clockwise) and vertically upward, respectively and u, v and w are velocity components respectively in r-, θ- and z- directions. When the Rossby number of the system is sufficiently small, the horizontal velocity components can be represented by geostrophic relations

$$u = u_g = - f^{-1} \partial h / (r \partial \theta), \quad v = v_g = f^{-1} \partial h / \partial r \qquad (2\ a, b)$$

where h is the depth outside the bottom Ekman layer with depth $De = \pi \sqrt{(2 \nu / f)}$ in which ν is a molecular viscosity. At the bottom $z = 0$, velocity and its vertical shear should vanish. Since $De << h$, the condition of a geostrophic flow at $z = h$ yields the horizontal velocity as

$$u = u_g - \exp(-\chi) (u_g \cos \chi + v_g \sin \chi) \qquad (3)$$

$$v = v_g - \exp(-\chi) (u_g \sin \chi - v_g \cos \chi) \qquad (4)$$

where $\chi = \pi z / De$. The equation of continuity with a source P per unit of volume is

$$r^{-1}\,\partial(ru)/\partial r + \partial v/(r\partial\theta) + \partial w/\partial z = P \tag{5}$$

Substituting (3) and (4) into (5), we have

$$\partial w/\partial z + \exp(-\chi)\,\sin\chi \left\{\partial u_g/\partial\theta - \partial(rv_g)/\partial r\right] r^{-1} = P \tag{6}$$

In the following, subscript g is dropped. At the surface $z = h$, the boundary condition for w_h becomes

$$w_h = u\,(\partial h/\partial r) + v\,\partial h/(r\partial\theta) + \partial h/\partial t \tag{7}$$

where $\partial h/\partial t$ is change of depth due to steady vertical motion of the free surface.

Integrating (6) with z from o to h and using (7), we have

$$u\,\partial h/\partial r + v\,\partial h/(r\partial\theta) + \partial h/\partial t + De\left[\partial u/\partial\theta - \partial(rv)/\partial r\right]/(2\pi r) = S \tag{8}$$

where $S = \int_o^h P\,dz$. Introducing a stream function defined by

$u = \partial\psi/(r\partial\theta)$, $v = -\partial\psi/\partial r$ into Equation (8), we have

$$(\partial h/\partial r)(\partial\psi/r\partial\theta) + De\,(2\pi)^{-1}\nabla^2\psi = S - \partial h/\partial t \tag{9}$$

This equation is similar to the equation for a wind-driven circulation derived by Stommel (1948) if $\partial h/\partial r$ and $S - \partial h/\partial t$ are considered as β and the wind stress, respectively.

Simulation of the β-effect by changing a depth may be achieved either by use of a distorted depth distribution or by taking advantage of rise of the free surface toward the rim due to a centrifugal effect. The former approach was taken by Ichiye and Plutchak (1962) for the Gulf of Mexico experiments. The latter approach was made by Stommel and others (1958) for their qualitative experiments in a pie-shaped basin. The equation of the

free surface is given by

$$h = h_o (1 + f^2 r^2 / 4 g h_o) \qquad (10)$$

where a is the radius of the basin and h_o is the depth at the center of the basin. Therefore, by taking suitable values of rotation speed and initial depth, the β-effect may be simulated for a flat bottom tank.

ACKNOWLEDGMENTS

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(ADDENDA)

A series of experiments including determination of stability of various kinds of jets and generation of vortices due to meanders are now underway by N. B. Plutchak and the author, using a rotating basin of 1.5 m diameter.

Explanation of Figures

Fig. 1	Positions of the Gulf Stream at the beginning and end of Operation Cabot after Fuglister and Worthington (1951). Isotherms of mean temperature (°F) of the upper 200-meter layer and belts of the "warm core" are shown west and east of 64° W, respectively.
Fig. 2	(A) Temperature (°C) at the 200-meter layer from Gulf Stream'60 (May-June, 1960) (B) Positions of the abrupt change in direction of the Gulf Stream and the position of the maximum gradient in the average 200-meter temperature [after Fuglister (1963)].
Fig. 3	Velocity distributions in Haurwitz and Panofsky's model (a, b and c) and in Kuo's model (d). (Numerical values in (d) are those used for Ichiye's calculation of Kr and Ki in Fig. 4).
Fig. 4	Roots of the frequency equations. I and II (H-P theory): $C_H = (c - U_1) / (U_{2\,3} - U_1)$, $d_2 = 100$, $d_3 = 20$ $d >> L$, $d_2 = 20$ (for I) and 200 (for II) all in nautical miles. K (Kuo's theory): $C_K = (c - U_o) / U_m - U_o)$, $U_m = 1.1$ (kt) $U_o = - 0.1$ (kt) (See Fig. 3-d) Suffices r and i denote respectively real and imaginary part.
Fig. 5	Diagram of the two-layer model used by Iida and Ichiye.
Fig. 6 (a)	Diagram of the two-layer ocean used by Duxbury: the equilibrium state is indicated by full lines, the disturbed state by dashed lines. The plane $y = 0$ represents a velocity shear boundary in the equilibrium state
(6b)	Graph of the negative real values of the relative frequency ν versus the relative wave number μ for $F^2 = 10.0$. The mode number represents the numbers of zeros of the function $Z(y)$ in the Region 1. (After Duxbury, 1963).
(6c)	Iwata's model of the two-layer system. Both sides of the uniform jet U are motionless and have constant depths D_2 and D_3.
Fig. 7	Semi-circle in complex velocity plane in which the phase speed for unstable baroclinic waves must lie.

Fig. 8	Trajectories of the constant absolute vorticity flow, after Rossby (1940) and Saint-Guily (1957). (a) I : $\theta_o = 45°$, II : $\theta_o = 90°$, III : $\theta_o = 117.5°$ (b) $\theta_o = 180°$
Fig. 9	Gulf Stream path observed on June 8, 1950 with a path calculated by applying Equation (23) to small segments A to L. After Warren (1963).
Fig. 10	Kinetic energy transfer from eddies to mean flow (solid line) and mean downstream velocity (dotted line) (a) The Kuroshio, after Ichiye (1957). (a-1): Between 135° E and 141° E. (a-2): between 141°E and 150° E. (b) The Gulf Stream after Webster (1961). (b-1): Off Onslow Bay. (b-2): Miami-CatCay. (c) The upper atmosphere after Starr (1953). (Dotted line: relative angular velocity.)
Fig. 11	The cross current eddy heat flux ($\overline{u'T'}$), the mean temperature ($\overline{T}$) and downstream velocity ($\overline{v}$) as a function of distance from the mean axis of the Gulf Stream. A positive value of eddy heat flux represents an offshore transport of heat (a) Cruise off Onslow Bay from May 20 to June 18, 1958. (b) Cruise off Jacksonville (30°N) from October 7 to November 15, 1961. (c) Cruise off Cape Hatteras during October, 1962 after Oort, 1963)
Fig. 12	Surface dynamic topography referred to 1000 d bar level of the Kuroshio (a and b, after Ichiye, 1957a) and bottom topography of the Kuroshio area (c, after Ichiyc, 1960) a) May-June, 1951 (No southward detour) b) May, 1955 (Southward detour between 135° and 140° E)

Explanation of Figures (cont'd.)

Fig. 13 (a) Southern limits of the Kuroshio between 135° and 140° E (Crosses denote the values determined from dynamic topography, and solid circles those from isotherms at 100-meter layer) and appearance periods of an isolated cold water (hatched bar) after Ichiye (1957).

(b) The positions of northern boundary of the Kuroshio along 144° E (a vertical bar) and the trend curve of coastal temperature varying with a period of 4.5 years (after Kawai, 1955)

Fig. 14 The positions of the axis of maximum current in the Kuroshio determined from GEK measurements and subsurface temperature distributions (a) 1956 to 1958 (b) 1959 (after Masuzawa, 1961)

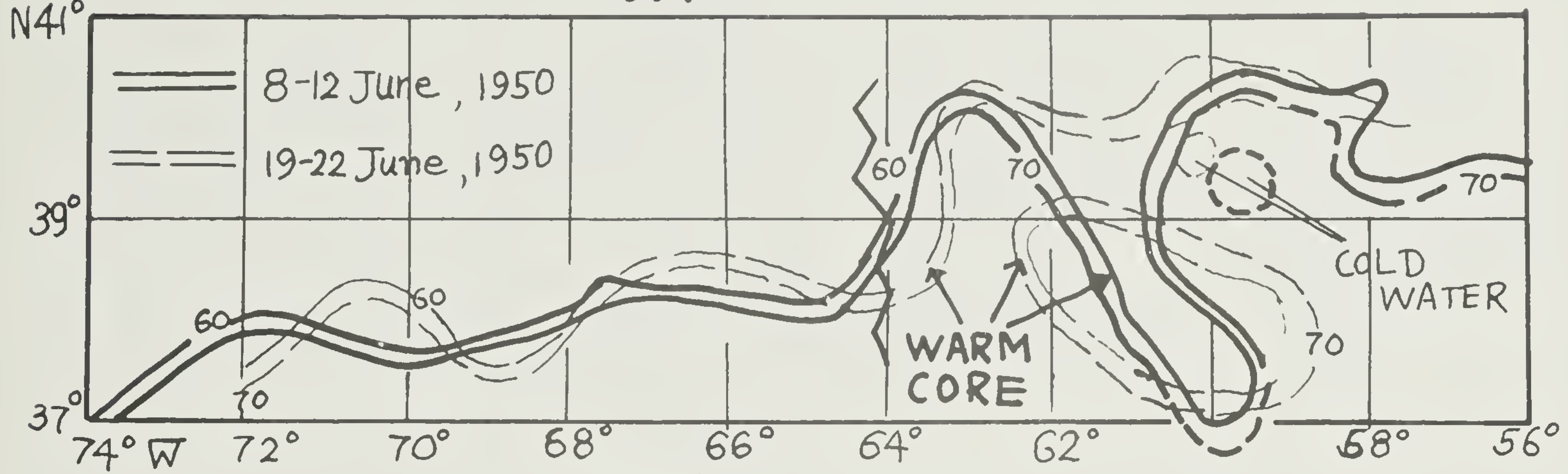
Fig. 15 The bathymetri chart (a) and temperature and current distributions (b)

(a) based on Monaco chart

(b) May-June, 1962. Thick arrows indicate surface currents between 1.0 and 1.5 knots and thin arrows between 0.5 and 0.9 knots measured with GEK. Isotherms at 100 meter depth are in centigrade.

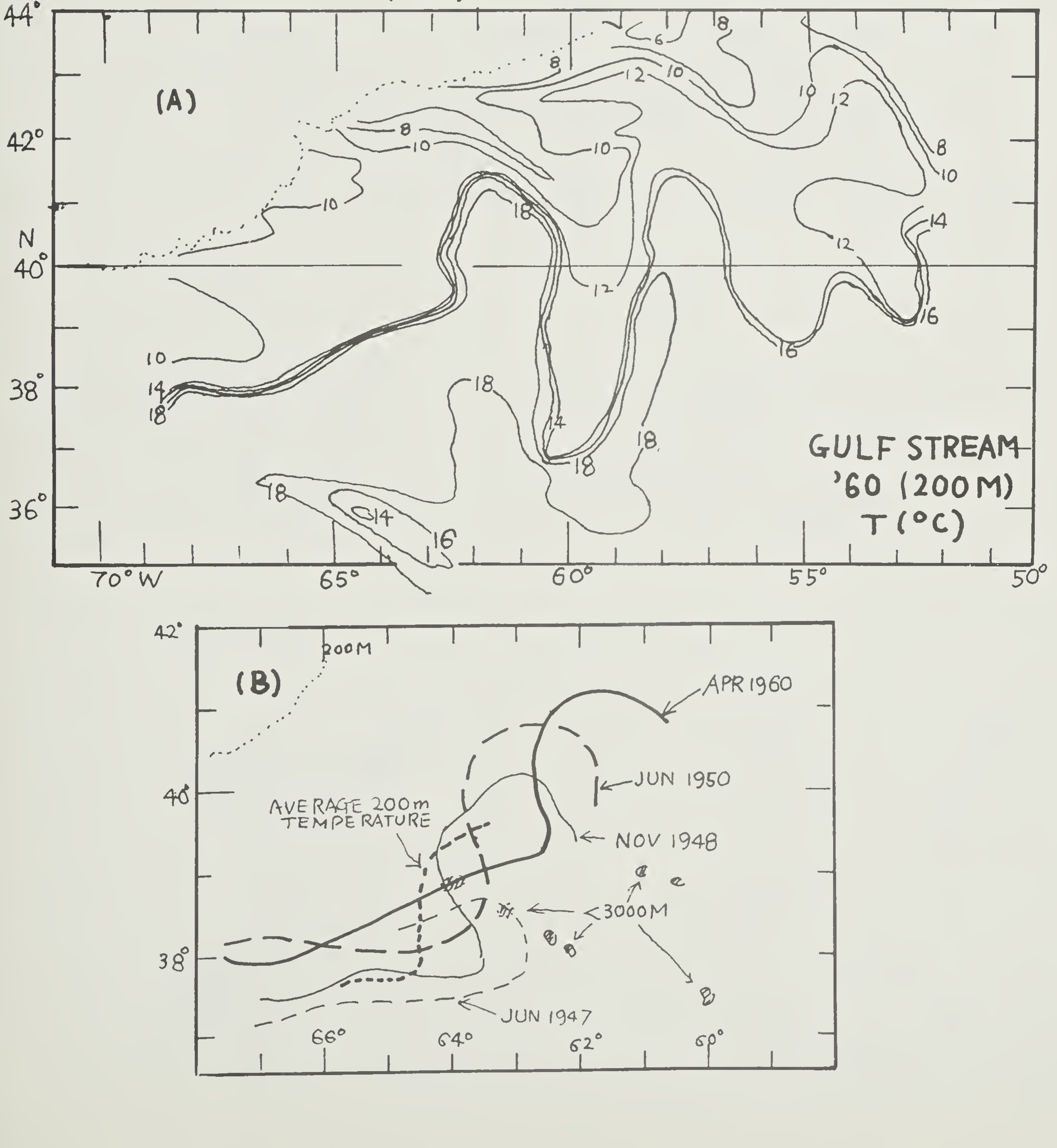
(after Japanese Hydrographic Office "Hydrographic chart" April-June, 1962).

FIG. 1



8-12 June, 1950
19-22 June, 1950
N41°
39°
37°
74° W
72°
70°
68°
66°
64°
62°
58°
56°
60
70
WARM CORE
COLD WATER

FIG. 2



(A)
44°
42°
N 40°
38°
36°
70° W
65°
60°
55°
50°
6
8
10
12
14
16
18
GULF STREAM '60 (200M) T (°C)
(B)
42°
40°
38°
66°
64°
62°
60°
200M
APR 1960
JUN 1950
NOV 1948
AVERAGE 200m TEMPERATURE
<3000M
JUN 1947

FIG. 3

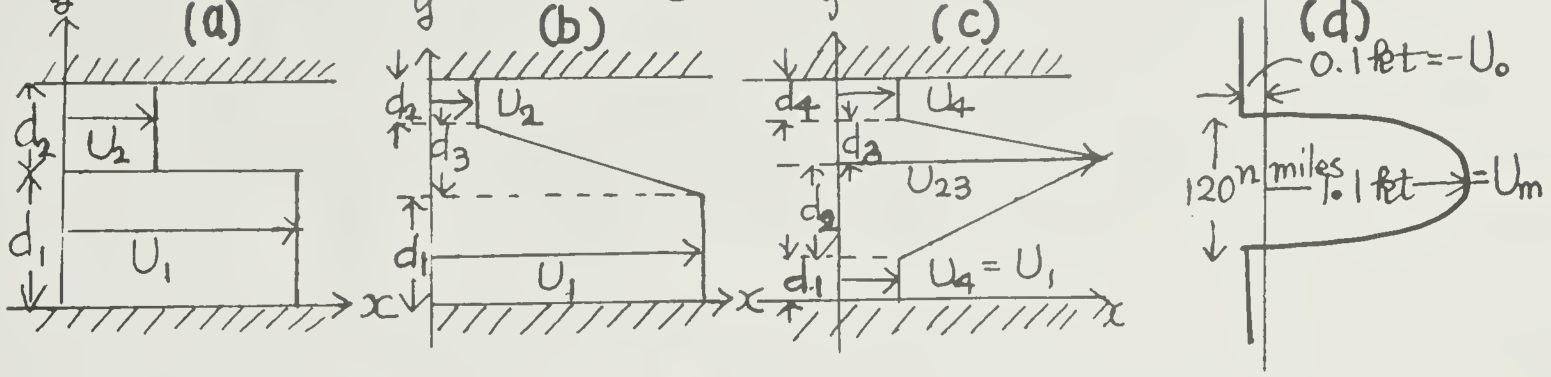


FIG. 4

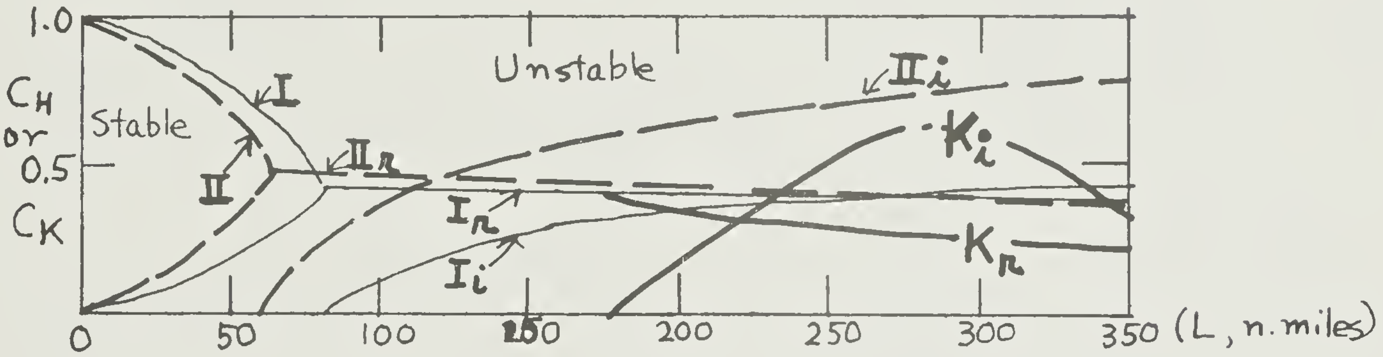


FIG. 5

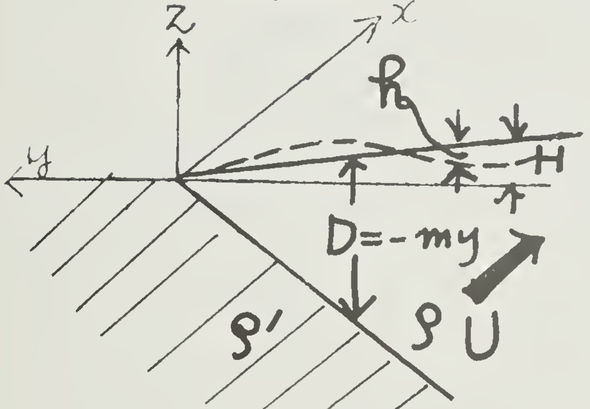


FIG. 6
(a)

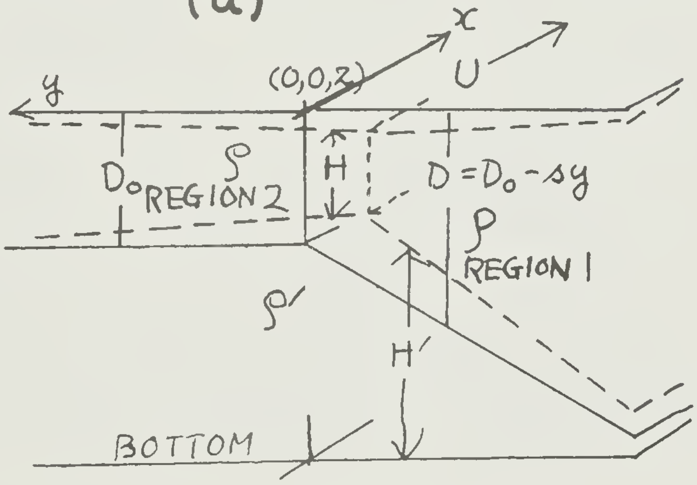


FIG. 6 (b)

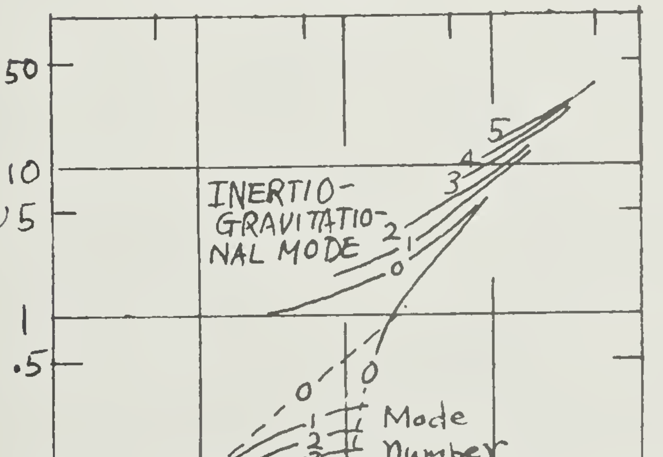


FIG. 6 (C)

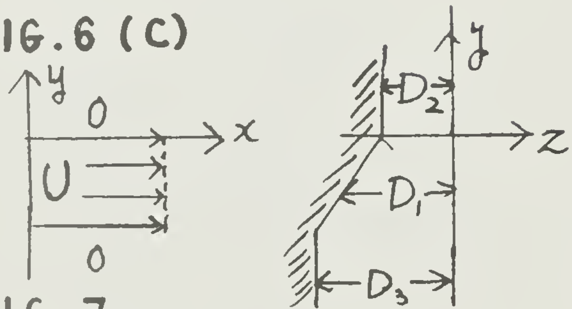


FIG. 7

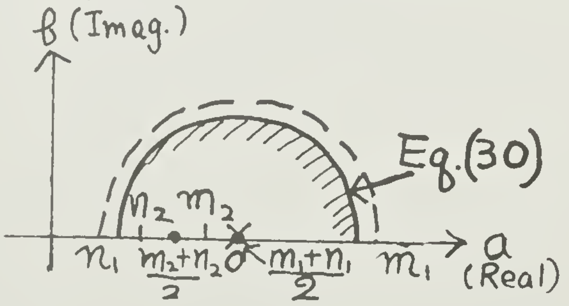


FIG. 8

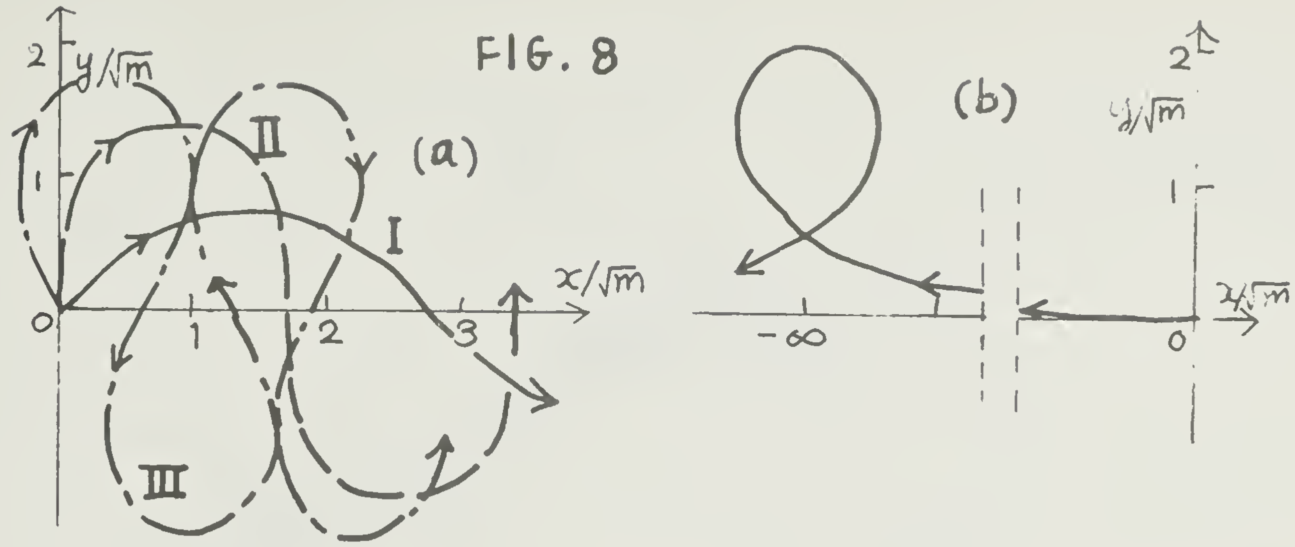


FIG. 9

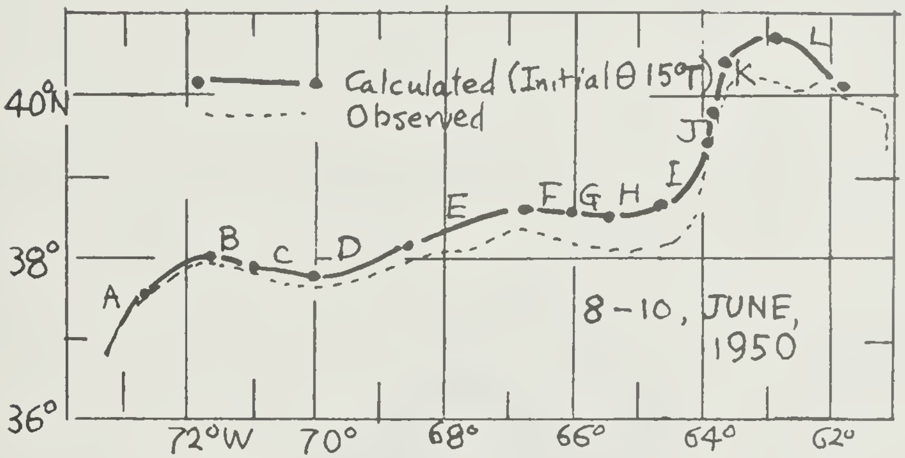


FIG. 10

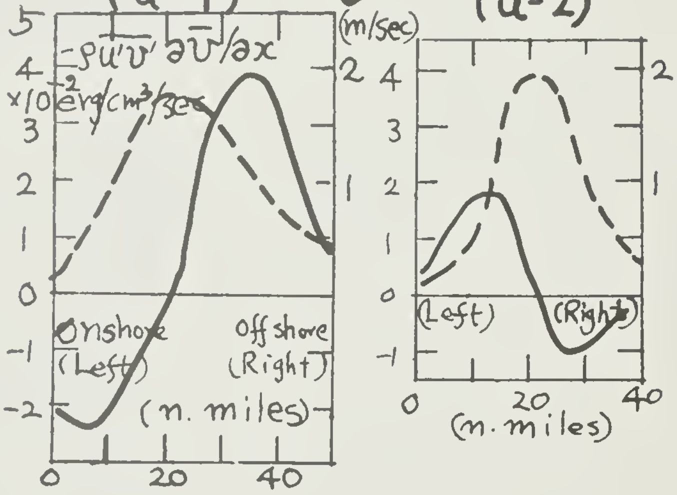


FIG. 10

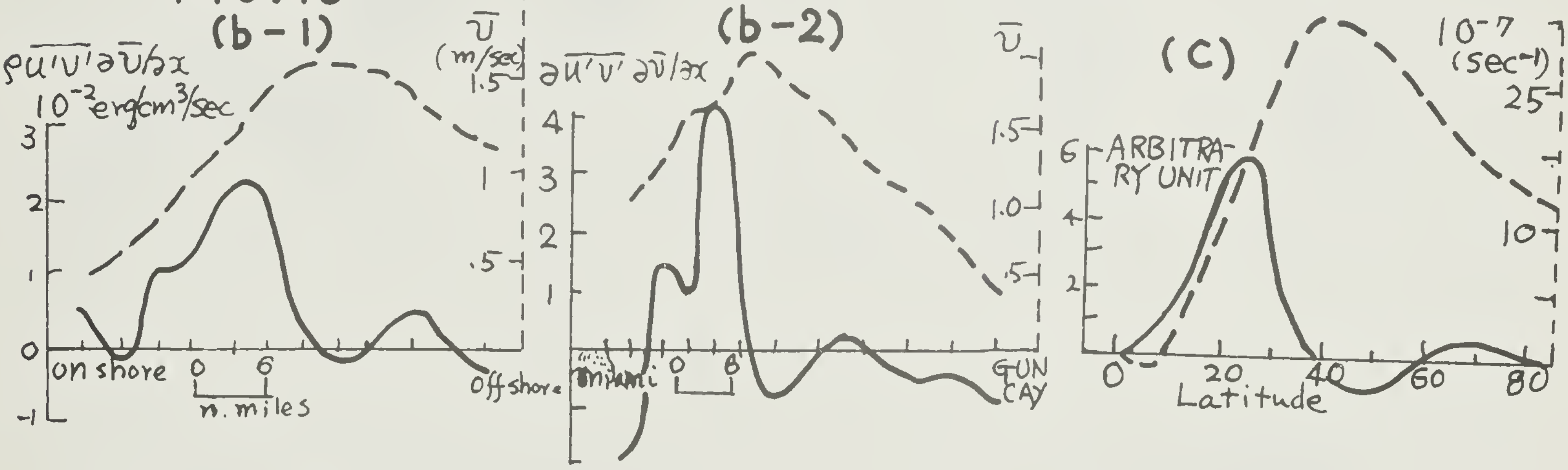


FIG. 11

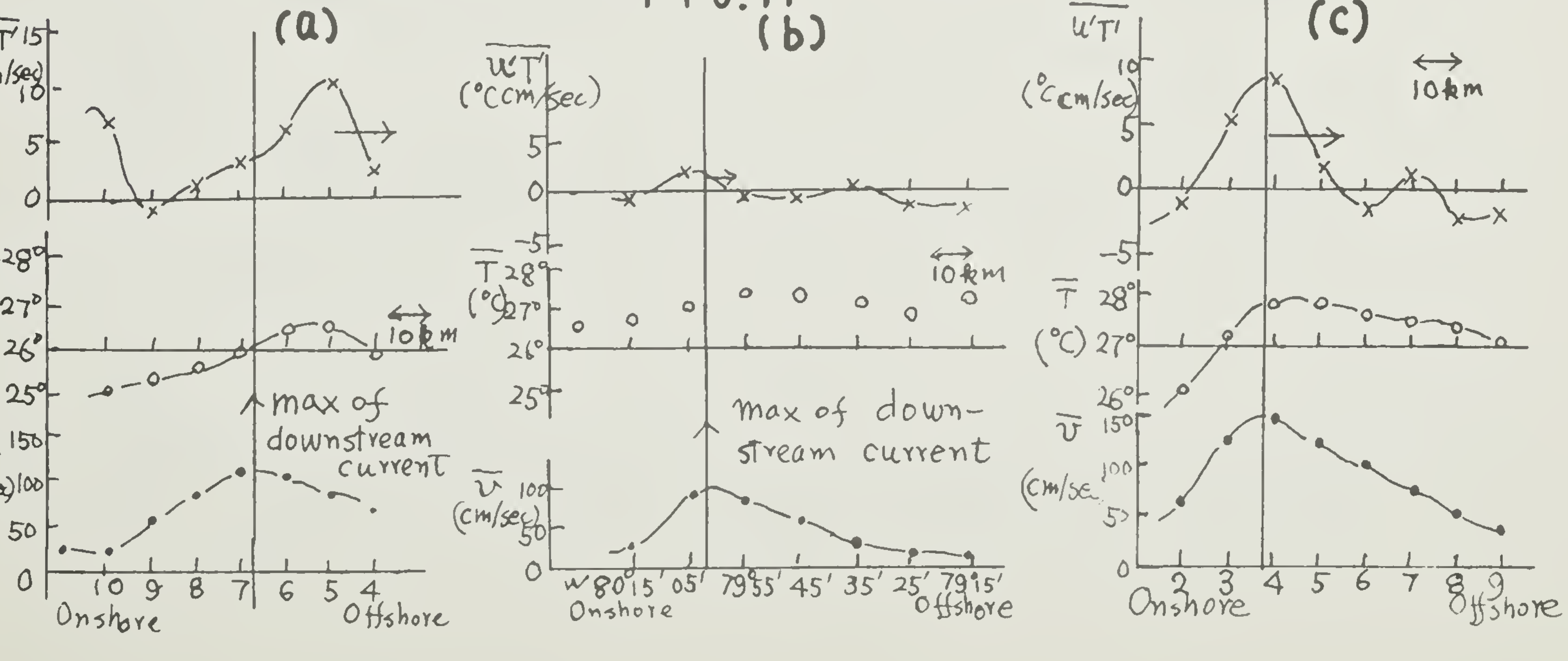


FIG. 12

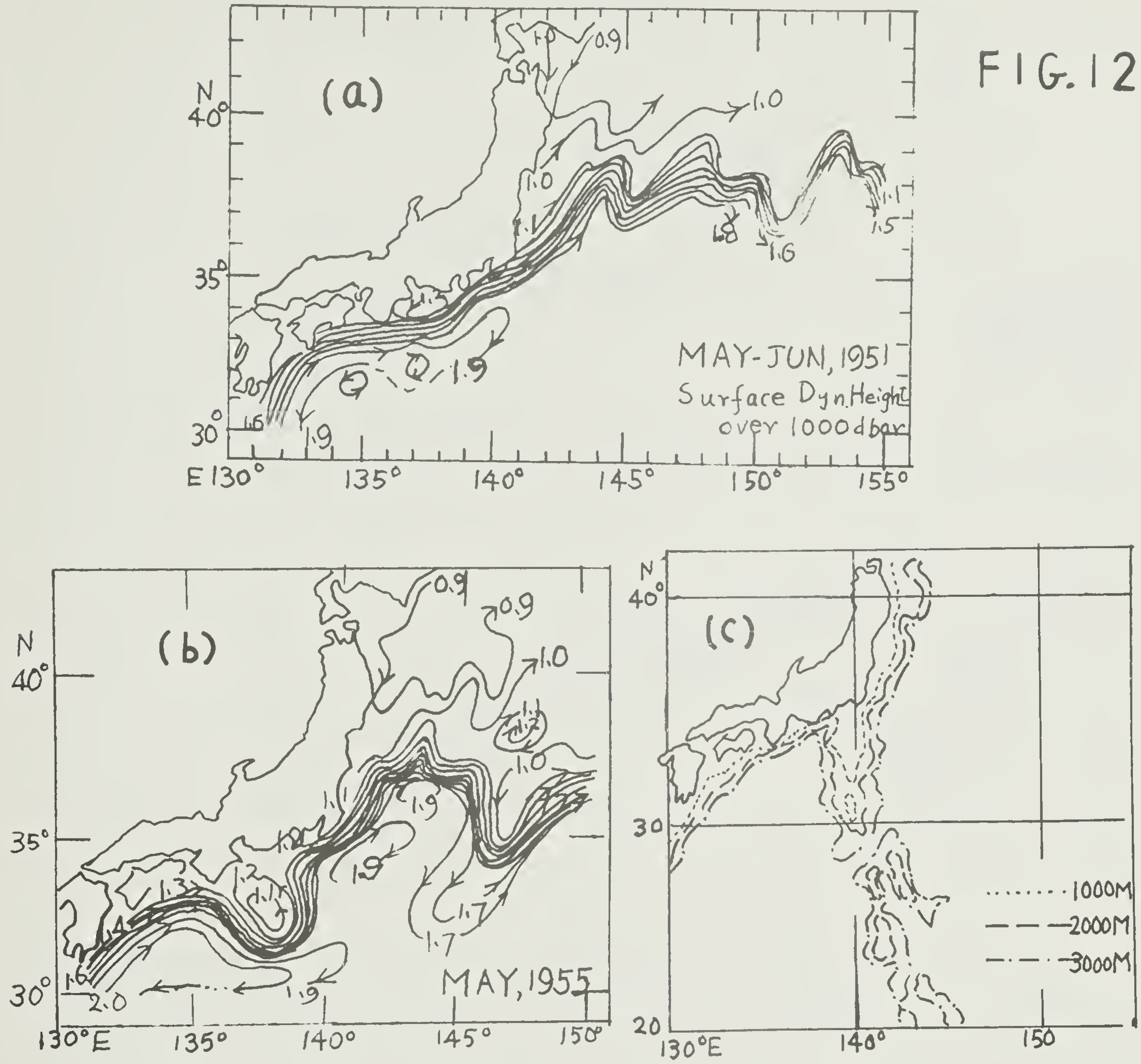

(a)
N
40°
35°
30°
E130°
135°
140°
145°
150°
155°
0.9
1.0
1.1
1.5
1.6
1.9
MAY-JUN, 1951
Surface Dyn.Height over 1000dbar
(b)
N
40°
35°
30°
130°E
135°
140°
145°
150°
0.9
1.0
1.1
1.2
1.3
1.4
1.6
1.7
1.9
2.0
MAY, 1955
(c)
N
40°
30
20
130°E
140°
150
1000M
2000M
3000M

FIG. 13

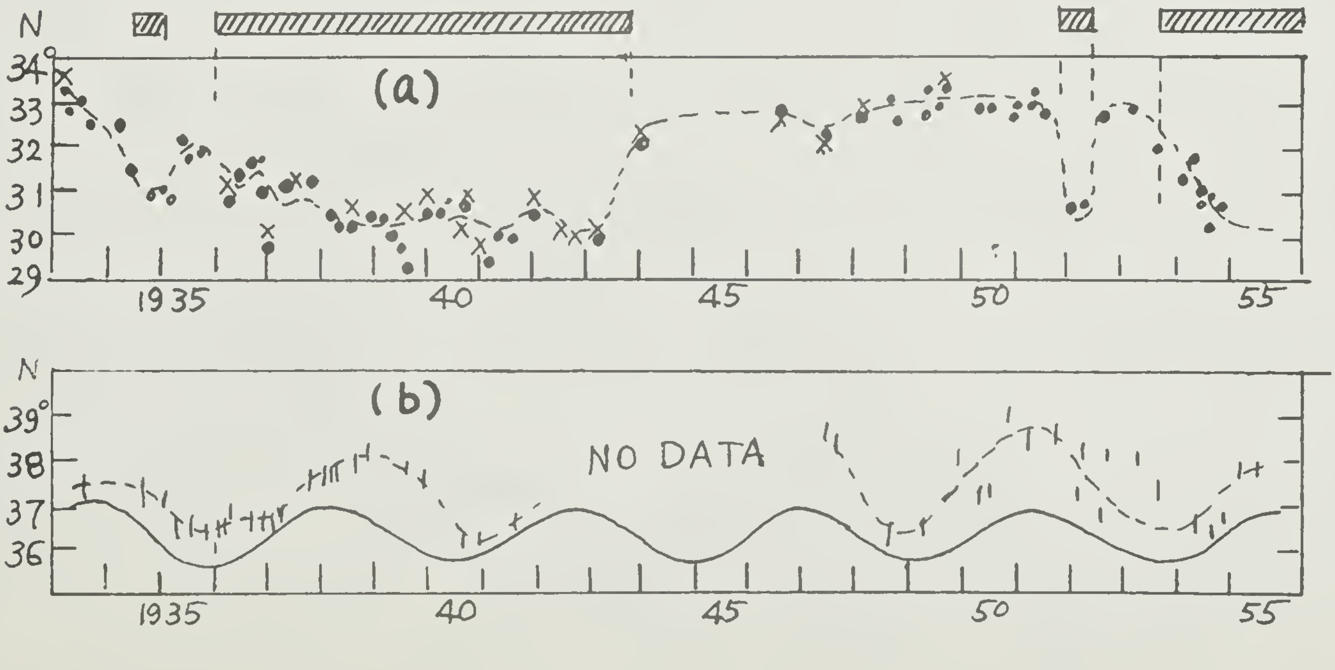

(a)
N
34°
33
32
31
30
29
1935
40
45
50
55
(b)
N
39°
38
37
36
NO DATA
1935
40
45
50
55

FIG. 14

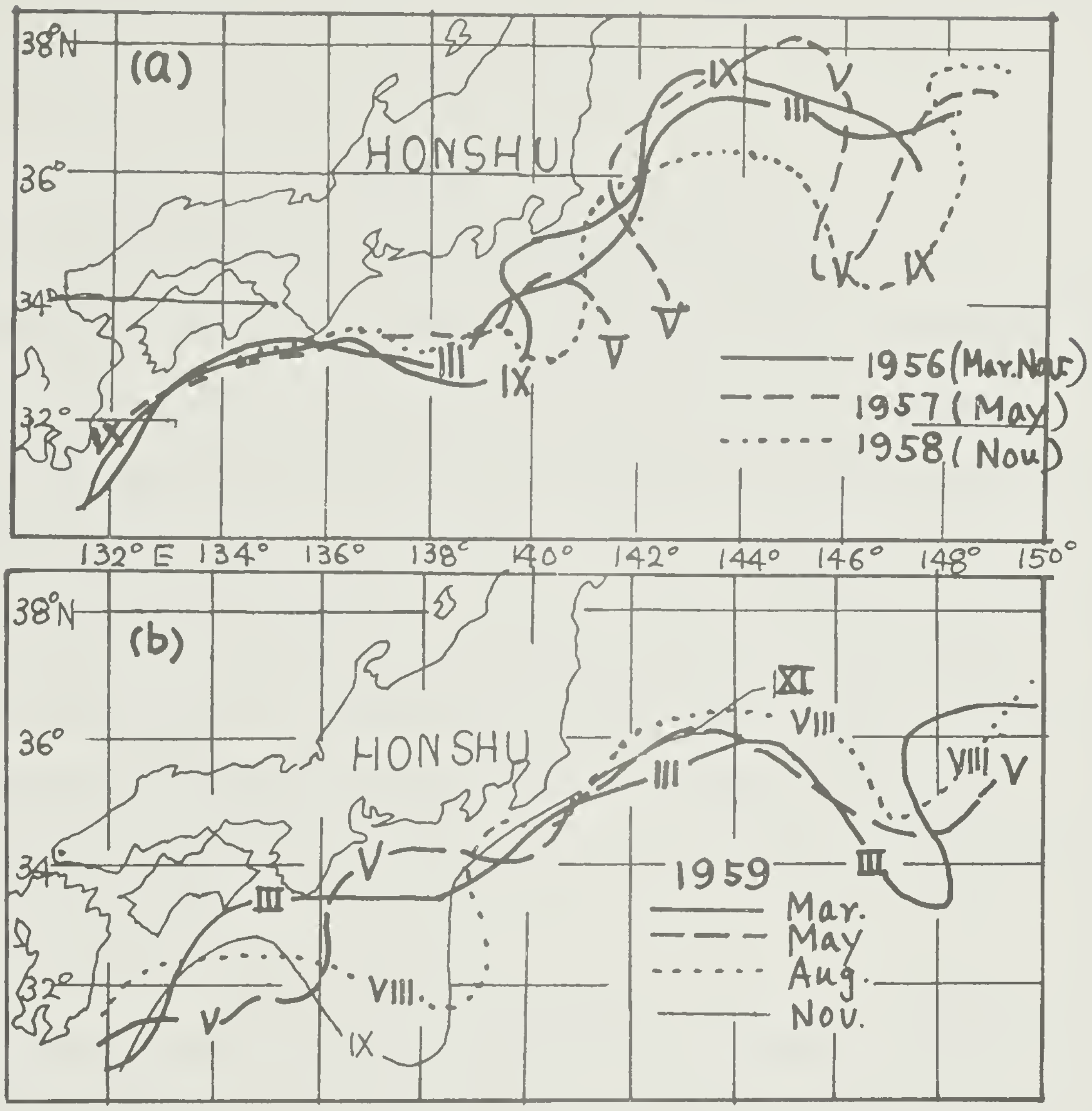
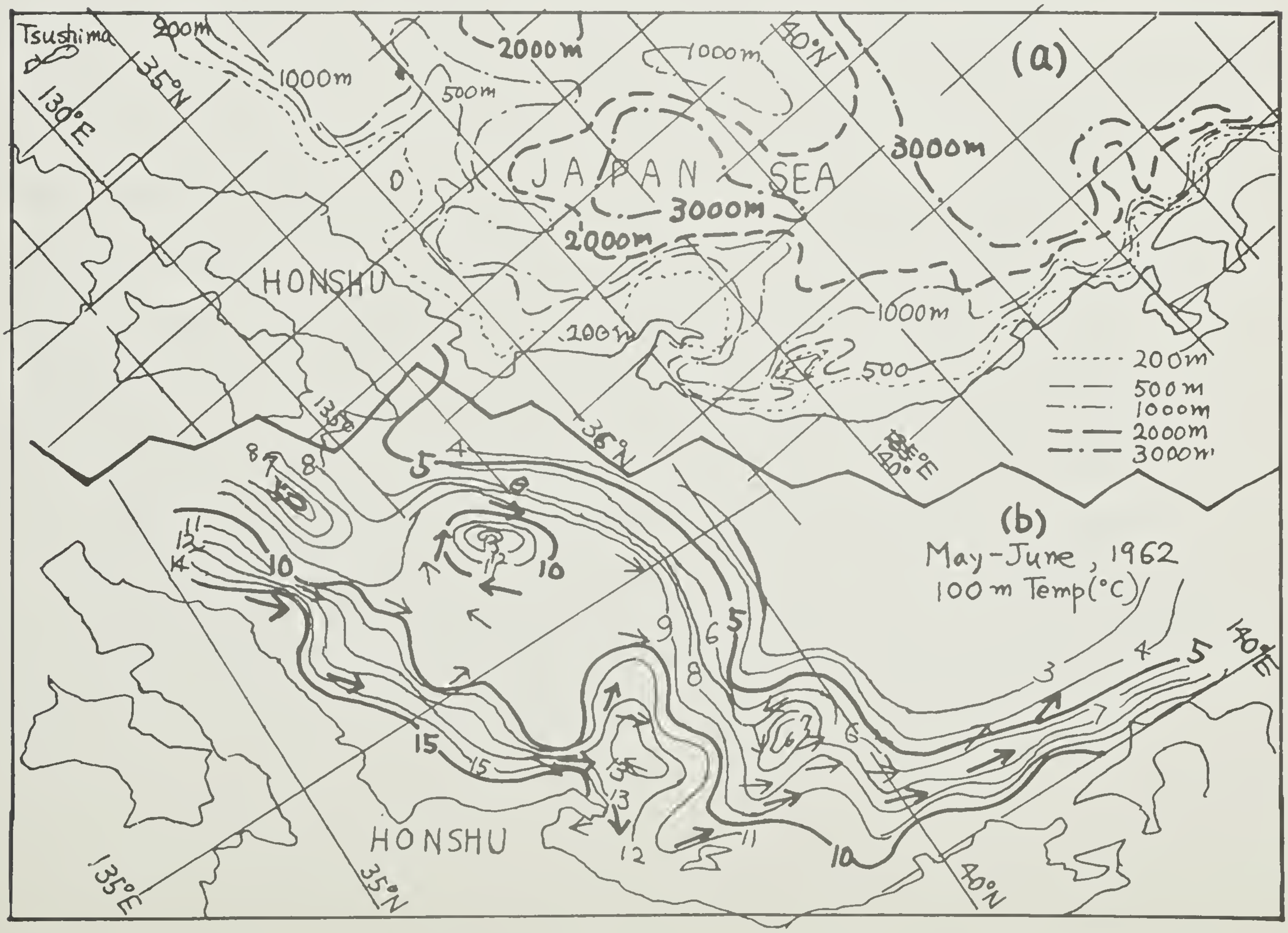

(a)
38°N
36°
34°
32°
HONSHU
IX
III
V
1956 (Mar. Nov.)
1957 (May)
1958 (Nov.)
132° E
134°
136°
138°
140°
142°
144°
146°
148°
150°
(b)
38°N
36°
34°
32°
HONSHU
XI
VIII
III
V
IX
1959
Mar.
May
Aug.
Nov.

FIG. 15


Tsushima
200m
1000m
130°E
35°N
2000m
500m
1000m
40°N
(a)
JAPAN SEA
3000m
2000m
HONSHU
200m
1000m
500
200m
500m
1000m
2000m
3000m
135°E
36°N
140°E
(b)
May-June, 1962
100 m Temp (°C)
8
10
11
12
14
15
13
5
4
9
6
3
HONSHU
135°E
35°N
40°N
140°E

APPENDIX

Since many examples for dynamics of the meanders in the text are taken from the Kuroshio, its behavior is described briefly. Hydrographic surveys of the Kuroshio after the World War II were mostly limited north of 25°, although the area from the east of the Philippine to the north was frequently surveyed before 1940. On the other hand, the area south and east of Japan between 130°E and 155°E was surveyed at least four times a year by five to eight research vessels almost synoptically since 1950 and thus seasonal changes and meanders of the Kuroshio are well known. The historical development of studies on the Kuroshio in Japan was summarized by Uda (1964).

In order to show somefeatures of the Kuroshio near Japan, two charts of surface dynamic topography relative to the 1000 d-bar surface in May-June, 1951 and May, 1955 are reproduced in fig.2 from Ichiye's paper (1957). In this figure bottom topography is also shown. It is seen immediately that the current pattern showed two different types to the south of Japan between 136° and 140°; one type with a large cyclonic detour around an isolated cold water as in 1955 and the other type with the flow close to the coast as in 1951. The separation of the Kuroshio from the coast occurs at around 36°N where the general coastline of Japan changes its direction from ENE to almost due north. Immediately after leaving the coast, the Kuroshio usually makes a weak anticylonic detour to the north between 142° and 146°E as a first half wave length of meanders. The sea to the east of 150°E was surveyed only occasionally and multiple currents seem to occur as in the Gulf Stream.

APPENDIX - 2

The southern detour of the current is peculiar to the Kuroshio and has no counterpart in the Gulf Stream. It lasted for several years, although the path of the flow changed in some degrees during its occurrence. There are many speculations on the cause of this detour but none of them is conclusive. However, since the detour occurs just upstream of the Bonin Ridge which also has no counter part in the Gulf Stream area, this ridge may be a main cause for the detour. In Fig.13 the latitudes of the southern most limit of the Kuroshio between 135° and 140° were determined from temperature distribution at 100m and dynamic topography as an indication of the extent of the southern detour and the result is plotted with comments on existence of a cold, cyclonic vortex (Ichiye, 1957). Also in this figure were plotted the northern limit of the Kuroshio along 144°E determined from temperature profiles and the schematic trend of the water temperature measured at three coastal stations along the northern coast of Japan (Kawai, 1955). It appears that the southern limit changed with occurrence of the cyclonic eddy with period of eight to ten years while the northern limit fluctuated with much shorter period of four to five years. Therefore in some periods of years the southern and northern limits were much apart while in others they were close together. A qualitative explanation of such coupling and decoupling process was discussed by Ichiye (1957), Moriyasu (1963) published series of charts showing the axis of the Kuroshio between 131° and 140°E and cold water mass accompanying with its cyclonic detour from 1931 to 1961.

APPENDIX - 3

Masuzawa (1961) determined the path of the axis of the Kuroshio between 130° and 150°E from surface velocity measured with GEK and from subsurface temperature measured with BT from 1954 to 1959. His results are shown in Fig. 14. Except in 1959, the path west of 136°E was almost constantly close to the coast and did not show much fluctuation. The southward detour between 136° and 140° started in the middle of 1953, disappeared in 1956 and reappeared in 1959. The meanders east of 145°E fluctuated more rapidly and particularly a cyclonic southward bend had much stronger curvature, large amplitude, and shorter duration than the one between 136° and 140°, suggesting that these meanders may be generated by an instability of a free jet. The northern limit of the Kuroshio between 142° and 140°E and the southern limit between 136° and 139°E are plotted also by Masusawa (1961). He showed that shift of the northern and southern limits was almost in the same direction until 1959, though the northward shift of the northern limit preceded the shift of the southern limit by about a year. The occurrence of the southern detour in 1959 was discussed by Moriyasu (1961) as four successive stage based on temperature at 100m and GEK data collected by different cruises between May 10 and June 9. In the first stage, May 10 to 14, the cold eddy of about 200 km length in the east-west direction and 100 km width in the north-south direction with its center at 31°N and 133.5°E was first detected by its 18°C isotherm at 100 m depth, where the main current of the Kuroshio flowed along the 20° isotherm. In the second stage, May 17 to 20, this eddy moved eastward and its center was located at 32.5°N 135°E. In the third stage, May 22 to 26, the detour of the current between 134°E and 137°E was almost the same as in

the second stage but the upstream side west of 134°E was found closer to the shore than before. In the fourth stage from May 29 to June 9 the center of the eddy was located at 33°N 137.5°E, further eastward from the location in the third stage. The center of the eddy moved slightly eastward in August to 138°E as indicated in Fig.14.

The maximum surface velocities measured with GEK are averaged over a period between 1954 and 1959 by Masuzawa (1961) and between 1955 and 1963 by Uda (1961) for segments with different longitudes between 130°E and 155°E. These averged values are from 2.6 to 3.7 knots according to the former, while Uda's values are generally lower by 0.2 to 0.4 knots. But both authors' results indicated that the averaged maximum speed is larger between 135°E and 138°E and between 143°E and 146°E and smaller between 130°E and 135°E and between 149°E and 152°E. This might be related to the meanders of semi-permanent type, although their existence is masked by those with shorter periods. As for the vertical structure of a velocity field in the Kuroshio, many authors disagree in their selection of the level of no motion. The consensus is that the current at 1000m still flows in the same direction as the surface current with speeds 9% to 30% of the surface velocity for the depths of no motion of 1200m and 3000m, respectively. (The 1200m depth was determined by Ichiye (1960) from comparison of GEK measurements with the dynamic calculation and the 3000m depth was obtained by Nan'niti (1962) with direct current measurements using neutrally buoyant floats.)

APPENDIX - 5

Another interesting feature of the Kuroshio which has no counterpart in the Gulf Stream is its branch current in the Japan Sea, usually called Tsushima Current. The temperature distribution at 100m depth and surface currents measured with GEK during May-June, 1962 are shown in Fig.15. This current is supposed to be separated from the Kuroshio at the latitude of 30°N and between 127° and 130°E but it is not distinguished as a strong boundary current up to the northeast of the Tsushima Straits, owing to the shallow depth and complicated coastlines. Inside the Japan Sea, the main part of the current flows close to the coast with a maximum speed of 1 to 1.5 knots and with a width of 50 to 100km. It grows rather stronger as it flows northeastwards and at the latitude of 41°N the greater part of the current flows out to the Pacific side through the Tsugaru Straits. The remaining part of the current flows further north and finally it is dissipated by mixing with the cold water. Between 35°N and 40°N the current usually produces two anticyclonic vortices of dimensions of the order of a few hundred kilometers to the left hand side (offshore). On some occasions the main flow detours clockwise around one or two of these vortices and thus only a fraction of the total transport moves close to the coast. It is unknown whether such a detour is seasonal or secular or how these two eddies change their extents and locations. Hydrographic data indicate that the current is geostrophic but its depth is rather limited to about 300 meters. Since this current has its coastal boundary to the right hand side the stabilizing effect of the coast is not the same as in the Gulf Stream and the Kuroshio. Also its thermohaline structure is somewhat different from the latter currents, since a permanent thermo-cline of the Tsushima Current is located between 100 and 150 meters and the temperature below 300 meters is almost uniformly 0 to 1° centigrade.

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13. ABSTRACT

Observed data of meanders of the Gulf Stream including those obtained in Operation Cabot and Gulf Stream '60 are described. Unstable long waves at lateral boundaries of barotropic currents or in a uniform current of a two-layer ocean are discussed from theories of Haurwitz and Panofsky or of Stommel, respectively. Theories on long waves in a shearing current and general criteria on stability of a jet in a two-layer system are reviewed. Rossby waves as a cause of meanders are discussed. Meanders of finite amplitudes are explained from various applications of the principle of constant absolute vorticity. Cross current eddy momentum and heat transfer are discussed. The use of a rotating tank for testing theories of meanders is explained. Observed facts of meanders of the Kuroshio are also presented.

14. KEY WORDS

Hydrodynamic theories on meanders of the Gulf Stream and similar western boundary currents are critically reviewed.

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